An Empirical Investigation of Efficiency and Price Uniformity in Competing Auctions *

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Abstract

This paper investigates empirically a number of hypotheses that are related to efficiency and price uniformity in online competing auctions for train tickets. The data set is ideal for analyzing competing auctions since each ticket is sold in a separate auction and all auctions with identical tickets start and end at exactly the same time. The results unambiguously demonstrate that there is a strong relationship between efficiency and price uniformity on the one hand and the number of submitted cross-bids on the other hand. These findings are in line with what theory suggests.

JEL Classification: D40; D44.
Key Words: Competing auctions; Cross-bidding; Efficiency; Price uniformity.

1 Introduction

The popularity of online auctions has motivated a recent and substantial amount of empirical and theoretical research. This research has not only provided a number of fundamental insights regarding characteristics and bidding behavior of online auction markets, it has also highlighted a number of potential problems in standard auction theory. For example, a common assumption in the auction literature is that there are no parallel auctions. This assumption does not describe online auction markets very well because these types of markets usually have a number of multiple simultaneously running auctions with identical items, i.e., a number of sellers that auction their identical goods in competing auctions. Competing auctions and characteristics of these have been analyzed by e.g. Anwar et al. (2006), Peters and Severinov (2006) or Moldovanu et al. (2008), whereas Ellison et al. (2004) consider coexistence and competition between different auction sites. This paper contributes to the existing literature by empirically assessing a number of hypotheses that

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are related to bidding behavior and performance of online auction markets with competing auctions. In particular, two theoretical predictions regarding efficiency and price uniformity from Peters and Severinov (2006) are investigated.

In online auctions, the bidding procedure is often automatic, meaning that bidders tell an automatic bidding agent or a proxy bidder their current willingness to pay for an item. As bidding progresses, the proxy bidder increases the bid by a minimal bid increment if a bidder is overbid and given that the current standing bid does not exceed the bidder’s current willingness to pay for the item. Provided that there is a single seller, the items are homogeneous and each bidder only demands one item, it is well-known that a second-price Vickrey (1961) auction generalization where each winning bidder pays the amount of the highest non-winning bid generates an ex post efficient outcome where it is a (weakly) dominant strategy for each bidder to state his true willingness to pay for the item.

In online auction markets with competing auctions, truthful bidding is no longer a dominant strategy even if the bids are placed by a proxy bidder and the bidders only demand one item. This was first observed by Peters and Severinov (2006). The intuitive reason underlying this observation is that if a bidder only bids once and bids her true willingness to pay, the bidder may end up paying a higher price than necessary because the bidder will forgo the opportunity to place bids in a competing auction with a currently lower standing bid. Despite the potential difficulties on online markets with competing auctions, Peters and Severinov (2006) demonstrated that if bidders adopt a cross-bidding strategy, then a (weak) perfect Bayesian equilibrium can be reached. In our context, a bid conforms to the cross-bidding strategy if it is placed in an auction with the lowest standing bid among the group of competing auctions i.e. a group of auctions for equivalent tickets (to be made precise below). Such a bid is called a cross-bid, and a cross-bidder is a bidder who only places cross-bids. As empirically observed by e.g. Anwar et al. (2006) and McCart et al. (2009), a non-negligible fraction of all bidders tend to bid in the auction with the lowest standing bid in conformity with the cross-bidding strategy.

The data set analyzed in this paper consists of 38 238 online auctions at the eBay-owned Swedish web site Tradera for Statens Järnvägar train tickets during the period September 1st – November 30th 2009. The data set can be divided into 6 046 groups of competing auctions. This is an ideal sample for analyzing competing auctions as each ticket is sold in a separate auction. Consequently:

i. the items in the competing auctions are completely homogeneous in the sense that bidders only know the route and the time of the departure and arrival, they do not even know what seats they are allocated when they bid; and,

ii. all competing auctions start and terminate at exactly the same time.

This data set is superior to the data sets in e.g. Anwar et al. (2006) and McCart et al. (2009) because their data does neither satisfy i. nor ii.

This article uses the above described data set to investigates two of the theoretical predictions in Peters and Severinov (2006). Namely, if all bidders adopt the cross-bidding strategy then:
a. the process generates an ex post efficient outcome\textsuperscript{1}; and,

b. the efficient set of trades occur at a uniform price.

We cannot expect, however, to fully confirm these theoretical predictions. One reason for this is that the ending time of an auction is not fixed in Peters and Severinov (2006) in sharp contrast to most of the auction mechanisms for online auctions including, e.g., the auction mechanism for train tickets investigated in this paper. Moreover, because a reported maximum willingness to pay for an item generally is not available at online auction web sites unless a bidder is outbid, it will be hard to evaluate prediction (a) because it requires that the true maximum willingness to pay for each bidder is known. What can be observed, however, is whether the bidders with the highest revealed valuations of the items in a group of competing auctions win one auction each. If this is not the case, there is an inefficiency because there exists a bidder who is not assigned a ticket even though he has revealed that he is willing to pay more for it than some other bidder who is awarded a ticket. Thus, one way to carry out a fair test of prediction (a) is by examining what the bidders reveal and which bidders ultimately win the train ticket auctions.

In summary, the theoretical predictions (a) and (b) are ideal results that will most likely never be realized in a group of competing auctions given the above observations. The main objective of this paper should therefore be regarded as an attempt to investigate how close real-life outcomes are to ideal theoretical predictions.

The analysis shows that 869 of the groups of competing auctions contain cross-bidders only. To make a rigorous and fair test of the theoretical predictions from Peters and Severinov (2006) only these 869 groups of competing auctions can be considered. In this subsample, the analysis demonstrates that 75.5 percent of the groups of competing auctions are (revealed) efficient and that prices are uniform in 30.0-57.7 percent of the groups of competing auctions depending on the definition of price uniformity. In the full sample, the analogous figures are 33.6 percent and 5.5-13.5 percent for efficiency and price uniformity, respectively.

The above observations are polar cases where one extreme considers all groups of competing auctions independently of bidding behavior (i.e. the full sample) and the other extreme only considers competing auctions where all bidders are cross-bidders. In addition, a closer look at the data reveals that a majority of the bids are cross-bids (69.9%) even though a majority of the bidders are non-cross-bidders. These observations combined indicate that it is relevant to investigate in more detail how the fraction of cross-bids in a group of competing auctions are related to efficiency and price uniformity. For this reason, the measures were recalculated for samples consisting only of groups of competing auctions with a minimum fraction of cross-bids equal to $x$, where $x$ is a number in the closed interval 0 to 1 (note that $x = 0$ and $x = 1$ represent the two polar cases described above). The

\textsuperscript{1}The theoretical prediction (a) only holds under the assumption that the seller sets the reservation price equal to the true costs. In this particular case, the reservation price is always set to 1 SEK (1 SEK = 0.101 EUR as of November 11, 2010) and it is likely that the reservation price 1 SEK reflects the true (marginal) cost for Statens Järnvägar for the tickets sold in this way.
analysis demonstrates a clear relationship between efficiency and price uniformity on the one hand and the minimum fraction of cross-bids on the other hand.

The main conclusions from the investigation is that the typical bidder does not satisfy the assumptions in Peters and Severinov (2006). Therefore, it is not unexpected that the real life outcomes are less than perfectly consistent with the theoretical predictions. However, if only groups of competing auctions with 100 percent cross-bidders are considered, the analysis shows that the theoretical predictions from Peters and Severinov (2006) are in large parts consistent with what can be observed in the data even though the bidding procedure as well as the proxy-bidding agent on Tradera differ slightly from the assumed counterparts in Peters and Severinov (2006).

The remaining part of the paper is outlined as follows. Section 2 provides some history and institutional features. Section 3 presents the data set and descriptive statistics. The hypotheses to be assessed are stated in Section 4. The main results are provided in Section 5. Section 6 concludes the paper.

2 History and Institutional Features

Statens Järnvägar (SJ, henceforth) was established in 1856. It is a publicly owned company that mainly runs passenger trains in Sweden. Up until 13 years ago it was only possible to buy SJ train tickets over the counter and by telephone, but in 1997 SJ started to sell tickets on their website. Ten years later, SJ introduced a new sales channel when they began to auction train tickets on the (eBay-owned) site Tradera which is the leading auction site on the Swedish market. The main reason for this was the desire to find alternative sales of train tickets. Ticket sales on Tradera were thus designed to complement the usual sales. The tickets sold at auctions, however, constitute a small part of SJ’s total ticket sales. In 2009, for example, only 150 000 tickets of a total of 29.7 million were sold at Tradera (0.5 percent).

The auction mechanism adopted by SJ resembles a second-price auction (Vickrey, 1961) and it works as follows. The auction starts no earlier than 48 hours before departure time and the reservation price of a train ticket is always set to 1 SEK. The bidders place bids by entering a maximum amount that they are willing to pay for the good. An automatic bidder (a so-called proxy bidding agent) then places bids on behalf of the agent using an automatic bid increment amount which depends on the current standing bid. The proxy bidder will only bid as much as necessary to make sure that the bidder remains the highest bidder (or meets the reservation price) up to the bidder’s maximum amount. Note, however, that a bidder’s maximum willingness to pay is kept confidential until it is exceeded by another bidder. The auction ends exactly six hours before departure time, and the winner is the bidder with the highest bid when the auction ends.

In the remaining part of the paper, a ticket will often be used synonymously with an auction as all actual SJ auctions are for one ticket only. Two tickets are considered to be equivalent if they are valid on the same train, or more precisely, since the train number is not available on Tradera, if the following conditions hold. The tickets are for trains:
i. departing at the same time;
ii. from the same station;
iii. arriving at the same final destination; and,
iv. for the same train type (X2000 or InterCity).

A group of *competing auctions* (GCA, henceforth) consists of all equivalent tickets.

### 3 Data and Descriptive Statistics

Using a tailored so-called web crawler, all available information for each of the 38,238 SJ train ticket auctions in the period September 1st–November 30th 2009 was gathered. In the data set, there are a total of 6,046 groups of competing auctions with an average of 6,325 auctions in each competing group (standard deviation² 3.187). Based on information from SJ’s web site (www.sj.se), the 38,238 auctions were divided into eight different train routes that all are assigned an ID number for practical purposes (see column 2 of Table 1). The number of auctions and the number of groups of competing auctions for the eight different routes can be found in Table 1.³

<table>
<thead>
<tr>
<th>Route</th>
<th>Route ID</th>
<th># auctions</th>
<th># GCA</th>
</tr>
</thead>
<tbody>
<tr>
<td>Göteborg – Malmö – Köpenhamn</td>
<td>1</td>
<td>6 649</td>
<td>906</td>
</tr>
<tr>
<td>Luleå – Kiruna – Narvik</td>
<td>2</td>
<td>1 631</td>
<td>349</td>
</tr>
<tr>
<td>Stockholm – Borlänge – Falun</td>
<td>3</td>
<td>6 594</td>
<td>847</td>
</tr>
<tr>
<td>Stockholm – Göteborg</td>
<td>4</td>
<td>8 892</td>
<td>1 415</td>
</tr>
<tr>
<td>Stockholm – Karlstad</td>
<td>5</td>
<td>5 234</td>
<td>672</td>
</tr>
<tr>
<td>Stockholm – Malmö – Köpenhamn</td>
<td>6</td>
<td>3 127</td>
<td>854</td>
</tr>
<tr>
<td>Stockholm – Sundsvall – Härnösand</td>
<td>7</td>
<td>3 723</td>
<td>633</td>
</tr>
<tr>
<td>Stockholm – Östersund – Storlien</td>
<td>8</td>
<td>2 388</td>
<td>370</td>
</tr>
<tr>
<td><strong>Total</strong></td>
<td></td>
<td>38 238</td>
<td>6 046</td>
</tr>
</tbody>
</table>

In total, 36,296 of the tickets in the sample were assigned to some bidder (94.92%). However, 5,368 of the auctions only had one bidder (14.04%). Consequently, these ticket were sold at the reservation price 1 SEK. The mean price of a ticket in the full sample is 124.4

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²In the remaining part of the paper, the standard deviation is denoted by $\sigma$.

³The direction of travel is ignored in Table 1. For example, Stockholm – Göteborg and Göteborg – Stockholm are considered to be the same route. Tickets that do not match a route exactly have been assigned a route based on the existing railroad network.
SEK (σ = 139.7) but the mean price is heavily route dependent (see column 2 of Table 2). For example, the mean price of a ticket between Stockholm–Malmö–Köpenhamn bought at Tradera is 200.3 SEK (σ = 173.1) whereas the mean price of a ticket between Göteborg–Malmö–Köpenhamn bought at Tradera is only 62.81 SEK (σ = 69.32).

In the full sample, there are a total of 266,971 bids placed by 66,216 unique bidders, and the average bidder in the sample places 1.982 bids per ticket (σ = 2.032). Regarding ticket statistics, there are on average 6.982 bids per ticket (σ = 6.351) and 3.522 unique bidders per ticket (σ = 2.246). As can be expected, the number of bids per ticket and the number of unique bidders per ticket is heavily correlated with the price of a ticket. In fact, the correlation coefficients are 0.930 and 0.871, respectively (both significant at the one percent level).

Table 2: Mean price and mean bid statistics for the eight routes (standard deviation in parenthesis).

<table>
<thead>
<tr>
<th>Route</th>
<th>Mean price (σ)</th>
<th>Bids per bidder and ticket (σ)</th>
<th>Bids per ticket (σ)</th>
<th>Bidders per ticket (σ)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>62.81 (69.32)</td>
<td>1.919 (1.914)</td>
<td>6.070 (5.643)</td>
<td>3.164 (2.058)</td>
</tr>
<tr>
<td>2</td>
<td>64.44 (78.30)</td>
<td>2.246 (2.444)</td>
<td>5.083 (6.015)</td>
<td>2.264 (1.810)</td>
</tr>
<tr>
<td>3</td>
<td>85.87 (74.69)</td>
<td>1.737 (1.635)</td>
<td>5.935 (4.643)</td>
<td>3.418 (1.856)</td>
</tr>
<tr>
<td>4</td>
<td>94.92 (111.3)</td>
<td>1.760 (1.623)</td>
<td>5.192 (5.089)</td>
<td>2.950 (2.095)</td>
</tr>
<tr>
<td>5</td>
<td>189.2 (180.8)</td>
<td>2.058 (2.149)</td>
<td>8.766 (7.381)</td>
<td>4.259 (2.584)</td>
</tr>
<tr>
<td>6</td>
<td>200.3 (173.1)</td>
<td>2.345 (2.555)</td>
<td>9.703 (7.684)</td>
<td>4.137 (2.286)</td>
</tr>
<tr>
<td>7</td>
<td>145.4 (138.6)</td>
<td>1.981 (1.932)</td>
<td>6.806 (5.825)</td>
<td>3.463 (2.111)</td>
</tr>
<tr>
<td>8</td>
<td>135.1 (136.5)</td>
<td>2.199 (2.309)</td>
<td>7.615 (6.815)</td>
<td>3.464 (2.056)</td>
</tr>
</tbody>
</table>

4 The Hypotheses

Before stating the hypotheses to be assessed, it is important to recall the precise definitions of a cross-bidding strategy and an ex post efficient outcome. Formally, a bid conforms to the cross-bidding strategy if it is placed on a ticket with the lowest standing bid among the tickets auctioned in group of competing auctions. Such a bid is called a cross-bid. A bidder follows the cross-bidding strategy in a group of competing auctions if all bids placed

4 This should be compared to the mean price of a ticket on the very same route bought at SJ’s web site which is around 700 SEK.

5 The standing bid in an auction at any given time is taken to be the smallest of the highest bid placed earlier, and the second highest bid placed earlier plus Tradera’s minimal increment, where identical bids are counted several times. If fewer than two bids have been placed then the standing bid is the reservation price. This interpretation does not necessarily correspond to what a bidder actually sees as Tradera’s auction histories are presented using a time resolution of minutes. The time it takes to place a bid may be also be non-negligible, e.g. if a bidder needs to authenticate first, or is experiencing network connection problems.
in this group of competing auctions conform to the cross-bidding strategy. Such a bidder is called a cross-bidder.

The outcome in a group of \( m \) competing auctions is defined to be \textit{ex post efficient} if the \( m \) buyers with the highest \textit{true} valuations of the items win one auction each. However, as described in the Introduction, truthful bidding is not a dominant strategy when competing auctions exist. Thus, it is likely that the bids and the observed maximum willingness to pay for the tickets do not reflect the true valuations.\(^6\) For this reason it will be difficult to test hypotheses related to ex post efficiency. However, what is true is that if, in a group of \( m \) competing auctions, the \( m \) bidders with the highest \textit{revealed} valuations of the items do not win one auction each, then there is an inefficiency in the sense that there exists a bidder who is not assigned a ticket even though she has revealed that she is willing to pay more for the ticket than some other bidder who is awarded a ticket. For this reason, we say that the outcome resulting for a group of competing auctions consisting of \( m \) auctions is \textit{revealed efficient} if the \( m \) buyers with the highest \textit{revealed} valuations of the items win one auction each.

As previously concluded in the Introduction, the data set is ideal for investigating competing auctions because all goods are completely homogeneous, all competing auctions end at exactly the same time and the reservation price for the seller reflects the true marginal cost. Thus, if all bidders adopt a cross-bidding strategy and if bidding is completely costless, the theoretical predictions in Peters and Severinov (2006) suggest the following two hypotheses:

\begin{itemize}
  \item \textit{Hypothesis 1}: The outcome of each group of competing auctions is (revealed) efficient.
  \item \textit{Hypothesis 2}: All tickets in a group of competing auctions have identical prices.
\end{itemize}

Because the theoretical predictions in Peters and Severinov (2006) assume that all bidders adopt a cross-bidding strategy, the above hypotheses can only be expected to be verified for the 869 groups of competing auctions in the sample where all bids conform to the cross-bidding strategy. Consequently, an obvious approach is to delete the remaining 5 177 groups of competing auctions from the sample and then test the hypotheses only for the 869 groups that satisfy all prerequisites. However, a detailed look at the data shows that the mean fraction of cross-bids per group of competing auctions is 0.699 (\( \sigma = 0.185 \)). Hence, even though a small fraction of all groups of competing auctions contain cross-bidders only, a majority of the bids in a group of competing auctions are cross-bids. This means that bidders tend to think in terms of cross-bids but they do not fully adopt a cross-bidding strategy.

To investigate in more detail how the “degree of cross-bidding” is related to the above two hypotheses, the data set will be reduced gradually in the following way. For each

\(^6\)Even if this is the case, it is not possible to obtain information regarding all bidders stated maximum willingness to pay by construction of the proxy bidding agent because the stated maximum willingness to pay is only accessible when a bidder is outbid. This in not only true for Tradera but also for all other online auction sites including e.g. eBay, uBid, Amazon and many more.
group of competing auctions, the fraction of cross-bids is calculated. A group of competing auctions with fraction $x$ of cross-bids is only included in the data set when considering groups of competing auctions with a fraction of cross-bids less than or equal to $x$. Figure 1 describes the size of the reduced data set depending on the specific selected minimum fraction. Note that in the extreme case when $x = 0$ all 6 046 groups of competing auctions (except the 50 groups where all tickets are unsold) are included in the data set and in the other extreme case when $x = 1$ only the 869 groups of competing auctions with 100 percent cross-bidders are part of the data set.

![Graph](image)

**Figure 1:** The fraction of the full data set used depending on the selected minimum fraction of cross-bids.

Given the reduction of the data set described above, the following weakening of Hypotheses 1 and 2 can be specified:

**Hypothesis 1A:** The fraction of revealed efficient groups of competing auctions increases with the minimum fraction of cross-bids $x$.

**Hypothesis 2A:** The fraction of groups of competing auctions with uniform price increases with the minimum fraction of cross-bids $x$.

Note that an empirical assessment of hypotheses 1A and 2A can be regarded as an assessment of how sensitive the theoretical predictions in Peters and Severinov (2006) are to varying degree of cross-bidding behavior.
5 The Results

In this section we assess and analyze the four hypotheses stated in the previous section; it is divided into two separate subsections where the first deals with the hypotheses that are related to efficiency and the latter concentrates on the hypotheses that concern price uniformity.

5.1 Efficiency

The data reveals that 75.5 percent of the 869 groups of competing auctions that contain only cross-bidders are revealed efficient (the corresponding number for the full sample of 5996 groups of competing auctions where at least one ticket is sold is 33.6 percent). At a first glance, this percentage number may seem surprisingly low considering that only bidders who have adopted a cross-bidding strategy are considered. However, a closer look at the proxy-bidding agent adopted by Tradera for train ticket auctions offers an explanation.

To see this, suppose that the current standing bid of all tickets in a group of competing auctions equals 2 SEK, and that a new bidder places a bid in one of the auctions in the group. Assume in addition that there is a bidder with a stated maximum willingness to pay of 3 SEK in this auction, and that new bidder has a high-enough willingness to pay to become the leader of the auction at the new standing bid of 4 SEK. Now, if no more bids are placed (e.g. due to time constraints, as discussed below), there is a bidder with a stated willingness to pay of 3 SEK who does not win a ticket while at the same time there are winners with stated willingness to pay equalling 2 SEK. Hence, the outcome of the auction is inefficient even though all bidders have placed bids on tickets with a lowest standing bid. Consequently, a conflict between the modeling assumptions in Peters and Severinov (2006) and real-life practice appears – in real-life situations it is not costless for bidders to monitor all competing auctions and bidders therefore rely on the proxy-bidding agent instead of manually submitting minimum bid increments. According to the bidding strategy in Peters and Severinov (2006), each bidder should “raise his bid as slowly as possible (p.223).” In the above example, this means that the new bidder should state a maximum willingness to pay of 3 SEK and test this in each auction in the group of competing auctions to see if it is possible to become a current leader of an auction at a standing bid of 3 SEK (instead of 4 SEK). This strategy, would however be too time consuming and therefore bidders rely on the proxy bidding agent.

Of course, possibly outdated web page data, technical difficulties (see footnote 5) and the fixed closing time of the auctions make it even harder for bidders to update their bids if they are outbid as in the above example. This is especially true in the presence of last-minute bids or so-called sniping. Many empirical investigations on Internet-based auctions (including e.g. Ockenfels and Roth, 2004; Roth and Ockenfels, 2002) have demonstrated that bidders tend to place a large proportion of their bids in the very last minute of the auction. Thus, if bidders adopt the cross-bidding strategy and therefore do not state their true willingness to pay, there is a risk that they will not have the time to update their bid if they are overbid in the very last minute of the auction. This study finds evidence that
Sniping is present at ticket auctions on Tradera. In the group of competing auctions with 100 percent cross-bidders, 6.0 percent of all bids were placed in the very last minute of the auction. The corresponding figure for the full data set is 7.3 percent.

We next note that (revealed as well as ex post) efficiency in a sense is a binary definition; an outcome is efficient or it is not. However, an outcome may be very "close" to being efficient but still be judged as inefficient. For example, consider two different groups of competing auctions that contain five auctions each. In the first group all the highest bidders except one wins the auction, and in the second group only one of the highest bidders wins. Obviously, both outcomes are inefficient, but most people would probably agree that the outcome in the first group is "more efficient" than the second. To investigate the efficiency results in some more detail and to get a measure of the magnitude of inefficiency we investigate the relative efficiency in a group of competing auctions. This is a score which is defined as:

\[
\text{Relative efficiency} = 1 - \frac{\text{number of highest bidders who do not receive a ticket}}{\text{the total number of bidders}}
\]

Thus a score of 1 corresponds to an efficient auction and a score of \(x\) less than 1 means that a fraction \((1 - x)\) of the total number of bidders should have won a ticket based on their bids alone. The mean score of relative efficiency for the 869 groups of competing auctions that contain only cross-bidders is 0.956 (\(\sigma = 0.086\)), and the corresponding number for the full sample of 5,996 groups of competing auctions where at least one ticket is sold is 0.888 (\(\sigma = 0.121\)). From this perspective, the auctions look almost efficient.

Figure 2 describes the relationship between the fraction of (revealed) efficient groups of competing auctions and the minimum fraction of cross-bids, and the relative efficiency score and the minimum fraction of cross-bids. As can be seen from the figure, both the efficient fraction of groups of competing auctions and the relative efficiency score are positively correlated with the minimum fraction of cross-bids. The number of efficient groups of competing auctions as well as the efficiency score appear to be almost constant up to the critical fraction of cross-bids 0.4. As seen from Figure 1 this is primarily an effect of the underlying data set not changing much up to that point. The efficient number of groups of competing auctions increases rapidly (and almost linearly) for fractions of cross-bids exceeding 0.4. The relative efficiency score also increases for fractions of cross-bids exceeding 0.4 but at a much slower rate. In summary, there is clear evidence that the fraction of cross-bids affects the probability of an efficient outcome and the probability of a high relative efficiency score leading to the conclusion that Hypothesis 1A is corroborated.

### 5.2 Price Uniformity

The mean price in the groups of competing auctions containing only cross-bidders is 25.08 SEK, and the prices are uniform in 30.0 percent of the groups. This should be compared with the sample of 5,996 groups of competing auctions where at least one ticket is sold where the mean price is 124.1 SEK and the prices are uniform in only 5.54 percent of the groups. A first observation is that this supports the findings in Anwar et al. (2006) where it
was demonstrated that cross-bidders pay a significantly lower price than non-cross-bidders.  

The reason for that the number of groups of competing auctions with uniform prices is relatively low can be seen by considering a simple example. Suppose as in Section 5.2 that the current standing bid of all tickets in a group of competing auctions equals 2 SEK, and that a new bidder places a bid. Assume next that the new bid leads to a new standing bid of 3 SEK for one of the tickets. Now, if no more bids are placed, the prices are non-uniform in spite of the fact that all bidders have adopted a cross-bidding strategy. This is due to the facts that the price for only one of the tickets was updated using the minimal price increment 1 SEK, and the observation that demand exceeded supply at price 2 SEK.

From the above, we conclude that it may be a too strong condition to require that all tickets in a group of competing auctions have identical winning prices when evaluating the hypothesis of price uniformity. Thus, when defining price uniformity it is reasonable to allow tickets to deviate in price by the minimal price increment prescribed by the proxy-bidding agent (e.g. 1 SEK in the above example). For this purpose, consider a group of competing auctions consisting of \( m \) tickets. Suppose that the winning prices of these tickets are gathered in the list \( \{p_1, p_2, \ldots, p_m\} \) and define \( p_* = \min\{p_1, p_2, \ldots, p_m\} \). The prices are then defined to be uniform if \( p_i \) belongs to the closed interval \( [p_*, p_* + \Delta] \) for each \( i = 1, \ldots, m \), where \( \Delta \) represents the minimum price increment prescribed by the proxy bidding agent at price \( p_* \). If this definition is employed, then 57.7 percent of the groups of
competing auctions containing only cross-bidders have a uniform price. The corresponding number for the full sample is 13.5 percent. Again, the groups of competing auctions with 100 percent cross-bidders stand out.

From the above we conclude that even with a more liberal definition of price uniformity, Hypothesis 2 is firmly rejected. A similar conclusion can be found in Anwar et al. (2006) even though no detailed analysis of price uniformity in competing auctions was provided. According to Peters and Severinov (2006), the finding in Anwar et al. (2006) could be due to the fact that the auctions in their sample did not end simultaneously. However, the competing auctions always end simultaneously in our sample. Still prices are non-uniform in 42.3% of the cases (using the more liberal definition). There are of course a number of different explanations for this finding, e.g., the construction in the proxy-bidding agent that causes prices to increase too much, the presence of sniping as described in Section 5.2 and the fact that the ticket auctions have a fixed ending time.

To get an idea of the magnitude of the price non-uniformity, the ticket price difference between the cheapest and most expensive ticket in each group of competing auctions was calculated. For the groups of competing auctions only containing cross-bidders, the mean difference between the cheapest and most expensive ticket was found to be 23.83 SEK ($\sigma = 46.25$). The corresponding number for the sample of 5,996 groups of competing auctions where at least one ticket is sold is 74.34 SEK ($\sigma = 71.20$). Hence, the mean price difference is significantly lower for the groups of competing auctions only containing cross-bidders. This is however quite natural considering the fact that the mean price in these groups of competing auctions is about one fifth of the mean price in the whole sample.

Figure 3 describes the relationship between the fraction of groups of competing auctions with uniform prices and the minimum fraction of cross-bids. Exactly as in the case of efficiency (i.e. Figure 2), there is a clear positive correlation between these two variables. This indicates that the fraction of cross-bids in a group of competing auctions de facto has a large influence on the probability of an outcome with uniform prices. Again, there is a critical point at fraction 0.4 in the sense that the number of groups of auctions with uniform prices increase approximately linearly for minimum fraction of cross-bids exceeding 0.4. This holds independently of which definition of price uniformity that is used. It is also interesting to note that the curve representing the more liberal definition of price uniformity increases more rapidly after the critical point 0.4 than the curve representing the standard definition. The implication of this is that the more liberal measure is more sensitive to cross-bids than the standard measure. In summary, we find quite strong corroborating evidence for Hypothesis 2A.

6 Conclusions

This paper has empirically investigated two of the theoretical predictions from Peters and Severinov (2006) regarding efficiency and price uniformity in competing auctions. To assess these predictions, data from SJ train ticket auctions from the auction site Tradera was gathered. We believe that it is difficult to find actual online competing auction data that
is better suited for an empirical test than the data used in this paper. However, as already stated in Section 1, we cannot expect to confirm these hypotheses since all underlying assumptions in Peters and Severinov (2006) will most likely never be satisfied in an actual real-life online auction and the data concerning the true valuations of all bidders is not accessible in general on online auction sites. For these reasons, this study should more be regarded as an attempt to assess how close real-life outcomes are to ideal theoretical predictions. However, despite the differences between real-life practice and the theoretical framework, this paper finds some evidence that supports the theoretical predictions in Peters and Severinov (2006), at least when the sample is restricted to groups of competing auctions that contain only cross-bidders.

There are a number of important implications from our findings. Most importantly, the competing Vickrey auction mechanism used by SJ and Tradera for online ticket auctions is not appropriate. There are many reasons for this. First, because inefficiencies are present, total revenue is not maximized. Second, because a bidder is not guaranteed to win an auction even if her bid is among the highest, the current mechanism is not attractive from a fairness point of view. The bidders are clearly aware of this and it has caused some irritation among them. For example, the following quotes are taken from the discussion forum on Tradera:⁷

Figure 3: The fraction of groups of competing auctions with uniform price depending on the minimum fraction of cross-bids.


"When SJ train tickets are auctioned on Tradera, it is not unusual that new bidders enter an auction and bid on the same ticket as me even if there are tickets on the same train without any bids...” (Alias: mattiph)

"It is common that a bidder only bids on one or two tickets, although there are many more tickets with the same departure time. In the end, it is all about luck, i.e. if you are lucky and happen to bid at a ticket with the lowest number of ‘active’ bidders...” (Alias: iréne_44)

Of course, the conclusion that the SJ train ticket auctions on Tradera are unfair is further strengthened by our results concerning price non-uniformity.

One of the authors of this paper (T. Andersson) presented the findings in this paper together with some additional and complementary results at the SJ headquarter in Stockholm. Our findings together with an in-house analysis at SJ resulted in that SJ has decided to change their auction mechanism for train tickets. Starting from 15 November 2010, SJ will auction their tickets in sequential auctions where the ending times are in two minute intervals. The main idea behind this new mechanism is that bidders that are outbid in an auction will get the opportunity to place bids on the very same ticket in an auction that ends two minutes later. There are indications in our data set that many of the loosing bidders will take advantage of this opportunity. For example, almost 30 percent of all bidders that are outbid in the last 20 minutes of an auction places a new bid in the group of competing auctions, and close to 25 percent of all bidders that do not win an auction place a bid on the next available train (that often has a departure time between 2–6 hours later).

References
