The Demand for Monetary Assets in the UK;

a Locally Flexible Demand System Analysis

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Abstract
This study provides strong empirical support for modeling the demand for monetary assets within a consumer demand framework. We estimate a linearised locally flexible almost ideal demand system, containing five monetary assets, over the period 1991Q4 to 1998Q4. Estimating the system in differences is a convenient method to account for possible non-stationarity in the data, a major concern for applied macroeconomists. All significant uncompensated own-price elasticities are negative. The compensated own-price elasticities are insignificant and the majority of the income elasticities are significant. Theoretical homogeneity and symmetry propositions are satisfied.
I. INTRODUCTION

In the theory of monetary aggregation, as set out by Barnett (1978; 1980; 1987), money is treated as a durable good rendering its owner a flow of services. This implies that the demand for monetary assets should be modeled using the same tools available to model the demand for any other good. These often highly non-linear demand systems differ greatly from the single-equation money demand specifications used in the traditional macroeconomic money demand literature. In that literature, empirical studies are often performed within a cointegrated vector autoregressive (VAR) model framework. An overwhelming majority of published work on money demand is based upon monetary aggregates that are constructed by simply adding various monetary asset quantities up to arbitrary levels of aggregation. Based on the assumption that monetary assets are durable goods, evidence from aggregation- and index number theory provide theoretically consistent methods to choose which assets that may be included in a monetary quantity aggregate and estimation- and parameter free methods to construct monetary quantity indices. These indices are simple sum indices only under very special circumstances. Furthermore, they are not stock measures. Instead, they are measures of the real monetary service flow generated by a group of monetary assets.

One line of research has been devoted to comparing the econometric performance of monetary services indices (MSI’s) and simple sum money in traditional macroeconomic models, in particular money demand models. Belongia (1996), for example, re-estimated several empirical models by replacing simple sum money with MSI’s and thereby significantly altered the conclusions that would have been reached in these influential US studies. Adding to this evidence, Konstantinou (2002) has shown that a stable money demand function exists for the US based upon a MSI that contains a broad range of monetary assets. Also considering US data, Lucas (2000, p. 270) states that “I share the widely held opinion that M1 is too narrow an ag-
aggregate for this period and I think that the Divisia approach offers much the best prospects for resolving this difficulty”. Outside the United States, evidence in support of MSI’s has been forthcoming from a broad range of countries. For example, Funding la Cour (2002) and Binner and Elger (2002) have replaced simple sum measures with MSI’s within the standard macroeconomic money demand framework. Using cointegration techniques, they have found stable money demand relationships with sensible coefficient estimates for Denmark and the UK respectively. An overview of the empirical evidence from eleven countries is given in Belongia and Binner (2000). Using the high/low road terminology in Barnett (1997), the use of Divisia money in standard money demand analysis can be considered a middle road approach. On the high road, the demand for money should “be modeled using the same tools that are reputable in modeling the demand for other goods and services” (Barnett (1997, p.1179)).

A growing body of literature has evolved studying the demand for money in models based upon microeconomic consumer demand theory. It should be noted that the usability of such estimates for monetary policy purposes has yet to be established. Serletis (2001, p. 264) argues that “a breakthrough from the current state of ‘interest target’ monetary policy back to the correct control of monetary quantities will be through demand systems. The research agenda is clear, and it starts with getting across the procedures and showing that elasticities make sense and the properties of the models are nicely neoclassical”. Work by i.e. Barnett and Yue (1988), Yue (1991), Fleissig and Swofford (1996) on US data and Drake, Fleissig and Mullineux (1999) on UK data has been based on Barnett and Jonas (1983) asymptotically ideal model (AIM). Fleissig and Serletis (2002) base their work on the Fourier flexible functional form using data from Canada and Fisher, Serletis and Rangel-Ruiz (2002) compare estimates form the translog, the AIM and the flexible Fourier using US data. Studies by i.e.
Elger (2001) and Collins and Anderson (1998) have been based on different versions of Deaton and Muellbauer’s (1980a; 1980b) almost ideal (AI) demand system. All models discussed in this paragraph are based upon flexible functional forms. In short, this means that the number of coefficients in the system is large enough to provide a reasonable approximation of the true underlying utility- or expenditure function. Flexible demand system specifications are inherently non-linear. This causes problems since the time series properties of the variables are often found to be unattractive. It is likely that the variables contain unit roots or that they are trended. Studying the exact properties is complicated and it is common to use univariate tests even though a system approach would be preferable. Nonstationarity has been extensively dealt with extensively in the macroeconomics literature, but less so in the microeconomic consumer demand literature. For a more general acceptance of the demand system approach towards studying the demand for monetary assets among applied macroeconomists, this issue must be more formally addressed. One of the major benefits of the AI demand system is that it can be linearised by assuming that the non-linear part of the system is a price index. This is not uncontroversial, but it is extensively used in empirical studies and it facilitates estimating the system in differences and thereby accounting for several sources of non-stationarity. The AI demand system (and on similar grounds the translog system) are sometimes criticized for being only locally flexible. The AI demand system is based upon a second order Taylor series expansion of the expenditure function. This means that (Deaton and Muellbauer (1980b, p. 74)) “it can only be guaranteed to be accurate in the locality of some point, at particular values of the price-income ratios” and, as noted by Yue (1991), the regularity conditions dictated by microeconomic theory are often violated in empirical studies based upon locally flexible functional forms. Models based upon global approximations of the indirect utility function, such as the AIM, do not suffer from these shortcomings, which is an advantage. One further problem in empirical demand system based money demand studies
is the fact that the data has been aggregated over consumers. Even though the AI demand system allows for a particular form of non-linear aggregation across consumers, one must ask to what extent regularity conditions can be expected to hold regardless of the type of demand system employed.

The major contribution of this paper is to model the demand for money in the UK over the period 1991Q4 to 1998Q4 using a linearised AI demand system estimated in differenced form. Differencing implicitly assumes that the variables are not cointegrated. Formally dealing with cointegration is not possible in the current study due to insufficient number of observations. Tests performed to study whether the variables are non-stationary are inconclusive and the decision to treat them as difference stationary is largely based on studying whether the residuals from the system in levels and differences are autocorrelated as well as visual inspection of the series (Appendix A).

The sample has been chosen following a study of weak separability by Binner, Elger and de Peretti (2002). In this study, six different monetary assets were found to be weakly separable from a number consumer goods (a broad definition) and leisure over the time period covered by the sample. Since the user costs for notes and coins and non-interest bearing bank deposits are identical, they have been treated as a single good in the current study. In several previous studies, weak separability has been assumed or the remaining consumption goods have been included in the system in an arbitrary fashion. For example, Yue (1991) creates one aggregate over all consumption goods that is included in the system jointly with the monetary assets. Weak separability (approximately) implies two-stage budgeting. Hence, consumers first allocate their spending over a broad range of goods and leisure. They then allocate their spending within each group given only the expenditure constraint from the first stage of the budgeting
process. This motivates estimating the demand system in two stages, see i.e. Edgerton (1997). Macroeconomic money demand models can be viewed as some form of first stage estimation. In this paper, the following utility structure is used:

\[ U(x, l, V(m)) \].

\(x\) contains non-durables, durables and services, \(l\) is leisure (the exact definitions are given in Binner, Elger and de Peretti (2002) and \(m\) contains the five different monetary assets\(^5\).

Estimation of the first stage of the demand system proved to be a difficult task. The coefficient estimates were either insignificant or not consistent with economic theory depending on what assets besides money that were included. Modeling leisure and consumer durables turned out particularly complicated. This is, however, a well-known finding and these problems are thoroughly discussed in Deaton and Muellbauer (1980b). Given the observed difficulties, only the second stage was estimated. In that stage of the analysis, we obtain significant coefficient estimates. The results support both homogeneity and symmetry of the substitution matrix. Rejections of both symmetry and homogeneity are common in models based upon aggregated data, so this must be considered a positive result. All significant uncompensated own-price elasticities are negative and all compensated price elasticities are insignificant. As would then be expected, the income elasticities are found to be strongly significant for the assets where the uncompensated price elasticities are significant.

This paper provides strong support for modeling the demand for money within a standard consumer demand framework. It is clear that much more research is needed to implement time series econometrics into the empirical estimation of demand systems. This is a chal-
lenging task, since most models proposed in the literature are non-linear. In order to win a more general acceptance for the demand systems approach, the bridge between the methods and theories employed by microeconomists and time series econometricians / macroeconomists must be narrowed.

The paper is organized as follows. In section two, monetary aggregation theory is introduced. The almost ideal demand system is presented and empirical methods are discussed. Section three describes the data and section four contains the empirical analysis. Section five concludes the paper.

2. THEORETICAL FRAMEWORK

A growing number of surveys exist that well cover the theory of monetary aggregation (see i.e. the papers by Anderson, Jones and Nesmith (1997a,1997b), Barnett, Fisher and Serletis (1992) and Fisher, Hudson and Pradhan (1993) as well as the textbook by Serletis (2001)) and the sections related to that theory are therefore deliberately kept short. The most relevant papers in the literature can be found in Barnett and Serletis (2000). Common for all studies within this theory is that they are based upon data that has been aggregated across consumers. Depending on the empirical application, this data is often converted to per capita data and the existence of a representative consumer is assumed.

User Cost

In the standard consumer demand theory, the price of a durable good is usually given by its equivalent rental price. A durable good is by definition a good that does not fully depreciate over a period. It provides its owner with a flow of services over that period. The market for durable goods can hence be viewed as a synthetic rental market. In the theory of monetary
aggregation the equivalent rental price is often denoted user cost. In the case that there is no risk, Barnett (1978, 1980) has shown that the nominal discrete time user cost of a monetary asset $i$ at time $t$ is;

$$\pi_{it} = P_i^* (R_t - r_{it})/(1 + R_t), \quad (2)$$

where $R_t$ is the nominal interest rate on a long run non-monetary investment, $r_{it}$ is the monetary asset's nominal own rate of return and $P_i^*$ is a true cost of living index. $R_t$ can be thought of as the rate of return on a completely non-monetary asset, e.g. human capital. A long run interest rate can be used as a proxy for $R_t$ or it can be constructed using an envelope approach (as promoted by e.g. Barnett and Liu (2000)). Using the theoretical framework developed by Barnett (1995) and Barnett, Liu and Jensen (1997), it is also possible to calculate the user cost under interest rate risk and risk aversion. The case of no risk (often denoted capital certainty) is shown to be a special case contained in the more general user cost formula under risk and risk aversion. Using (1) is strongly motivated by empirical findings. The risk adjustment of the user costs proposed in later studies are often found to be negligible, especially for more “traditional” monetary assets such as bank deposits (see i.e. Barnett, Liu and Jensen (1997) and Binner and Elger (2002)).

The Demand System

Deaton and Muellbauer (1980a, 1980b) derive their demand system from an expenditure function. Assuming that there are $k=1…N$ goods in the economy, the expenditure function, $e$, is defined as;

$$\ln e(u_t, p_t) = \alpha_0 + \sum_{k=1}^{N} \alpha_k \ln p_{kt} + \frac{1}{2} \sum_{k=1}^{N} \sum_{j=1}^{N} \gamma_{kj} \ln p_{kt} \ln p_{jt} + u_t \beta_0 \prod_{k=1}^{N} p_{kt}^{\beta_k}, \quad (7)$$
where $u_t$ is utility at time $t$, $p_{kt}$ is the price of good $k$ at time $t$ and $p_t$ is a column vector of prices at time $t$. Note that all equations in this section are written in deterministic form. Based upon the chosen functional form of the expenditure function, Deaton and Muellbauer (1980a) derive the following $i=1\ldots N$ equation system;

\[
s_{it} = \alpha_i + \sum_{j=1}^{N} \gamma_{ij} \ln p_{jt} + \beta_i \left( \ln x_t - \ln P_t \right), \tag{8}
\]

where $\ln P_t = \alpha_0 + \sum_{k=1}^{N} \alpha_k \ln p_{kt} + \frac{1}{2} \sum_{k=1}^{N} \sum_{j=1}^{N} \gamma_{kj} \ln p_{kt} \ln p_{jt}$ and $\gamma_{ij} = \frac{1}{2} \left( \gamma_{ij}^* + \gamma_{ji}^* \right)$. $P_t$ can be (approximately) viewed as a price index over all goods. The expenditure share of good $i$ at time $t$ is defined as $s_{it} = p_{it} q_{it} / x_t$, where $p_{it} q_{it}$ is the per capita expenditure on good $i$ at time $t$ and $x_t$ is total per capita expenditure at time $t$. $\alpha$, $\gamma$ and $\beta$ are coefficients that can be estimated empirically. It is possible to make (8) linear in the parameters by using the Stone price index as an approximation;

\[
P_t = \sum_{i=1}^{N} s_{it} \ln p_{it}. \tag{9}
\]

Much empirical research indicates that the residuals (see section 2.5, below) from both non-linear and linearised estimations of (8) are highly correlated. It has been common to attribute this to the fact that (8) is not likely to capture the dynamics of the demand system and that it is probable that consumption decisions depend upon habit formation, adjustment costs and other factors that can delay changes in consumer behavior. It has therefore been common to explicitly adjust the system to account for dynamics in consumption (see e.g. Allesie and Kapteyn (1991)). It is quite plausible that habit formation affects consumer demand, but the problem
may be even more complex. In the time series econometrics literature, autocorrelation (in combination with high $R^2$ is often interpreted as a sign of what Granger and Newbold (1974) referred to as spurious regression.

Dealing with potential non-stationarity in the data can conveniently be done by estimating the system in a differenced form. In doing so, potential cointegration is ignored and information on long run elasticities is lost. It is still possible to test for homogeneity and symmetry and to calculate short-run elasticities (see below). The linearised almost ideal demand system in first differences is:

$$
\Delta s_{ij} = \sum_{j=1}^{N} \gamma_{ij} \Delta \ln p_{ji} + \beta_{i} \Delta (\ln x_{i} - \ln P_{i}),
$$

(10)

where $P_{i}$ is the Stone index previously defined.

Micro-economic theory provides conditions that have to be satisfied in order for the system to be consistent with theory (see i.e. Edgerton et al (1996) or Deaton and Muellbauer (1980b)).

- **Adding up** follows from the budget restriction and the monotonicity of preferences. The representative consumer is assumed to spend all of his income.
- The demand functions are *homogeneous of degree zero*, implying $\sum_{j=1}^{N} \gamma_{ij} = 0 \forall i$. Zero degree homogeneity means that scaling prices and income by an arbitrary scalar does not alter the budget constraint or the utility function. There is no money illusion.
- Young’s theorem can be used to show that the elements of the substitution matrix are *symmetric*. Hence, in empirical estimates of (10), $\gamma_{ij} = \gamma_{ji} \forall i, j$. 

• The substitution matrix is negative semi-definite. All compensated own-price elasticities are non-positive (see i.e. Edgerton et al (1996, p. 61)).

It can be noted that adding up is automatically satisfied in the model under investigation. The reason is that the expenditure shares sum up to one (zero in first differences) by model construction.

Estimation Method

In empirical applications of the differenced linearised AI demand system, it is common to add an additive error term to (10). Let \( \varepsilon_t \) be a vector containing the errors at time \( t \). The errors are assumed to satisfy \( E(\varepsilon_t) = 0 \) (correct functional specification), \( E(\varepsilon_t, \varepsilon_t') = 0 \) (no autocorrelation) and \( E(\varepsilon_t, \varepsilon_t') = \Sigma \). A common feature in empirical demand system estimations is that the residuals are correlated across equations. Depending on the model specification, efficiency can be gained by incorporating the additional information contained in the covariance matrix into the system. There are numerous methods available for this purpose. The choice of method depends on which model is estimated and what hypothesis to test (see i.e. Edgerton et al (1996) and Greene (2000)).

Stacking the matrices of the individual linear regression equations in a convenient way, the following (compact) definitions may be used; Let the vector of coefficients to be estimated be denoted by \( \Gamma \). Define the stacked matrix of dependent variables as \( X \) and the stacked vector of independent variables as \( y \). \( \Sigma \) denotes the covariance matrix. Let \( V = \Sigma \otimes I \).

\[
\hat{\Gamma} = \left( X'V^{-1}X \right)^{-1} X'V^{-1}y
\]

is then an efficient generalized least-squares estimator (note that \( V^{-1} = \Sigma^{-1} \otimes I \)). The matrix of covariances is generally not known and one is hence dependent upon finding consistent estimators of it. Various feasible generalized least squares estimators
are available. One useful result is that ordinary least squares (OLS), seemingly unrelated regressions (SUR) and iterated seemingly unrelated regressions (ISUR) are identical if the equations in the system share the same explanatory variables and there are no cross-equation restrictions. The covariance matrix is estimated using OLS in SUR estimations. In ISUR estimations, it is estimated using an iterative procedure.

In the model under investigation, all equations share the same explanatory variables. OLS can therefore be used to estimate the system efficiently and to perform single equation tests. When tests for symmetry are performed, the restrictions are not identical across equations and ISUR is used. One useful result is that ISUR can be shown to yield maximum likelihood estimates if the errors are normally distributed. The benefit of using a maximum likelihood estimator follows from the fact that it is only possible to estimate \(N-1\) equations in the system. The reason is that the expenditure shares sum to one non-stochastically\(^8\), resulting in a singular covariance matrix. It is, however, possible to restore the unknown coefficients recursively. Maximum likelihood (ISUR) is said to be invariant, which means that the restoration of the unknown parameters is independent of which equation is removed for identification purposes. This can be considered an advantage.

III. DATA

The data used in this paper is quarterly covering the period 1991Q4 to 1998Q4, which gives a total of 29 observations (which gives 28 usable observations after differencing). The household sector holdings of the components included in the Bank of England MSI and their returns have been downloaded from the Bank of England Monetary and Financial Statistics division website, which is available at http://www.bankofengland.co.uk/mfsd/index.htm (Statistical Abstracts, Part 2). The bank of England MSI contains notes and coins (NC), non-interest-bearing bank deposits (NIBD), interest-bearing bank sight deposits (IBSD), interest-bearing
bank time deposits (IBTD) and building society deposits (BSD). For a further description of this data set, see Fisher, Hudson and Pradhan (1993). Since the user costs for notes and coins and non-interest-bearing deposits are identical, these assets have been aggregated into one good, non-interest-bearing assets (NI), as discussed in the introduction. Recent studies have proposed extending the list of assets to include national savings (NS) and certificates of deposit (see e.g. Drake and Chrystal (1997)). In the current study, only national savings have been added to the list of monetary assets provided by the Bank of England. Certificates of deposits are only available from 1986 to present and where thus excluded from the separability analysis by Binner, Elger and de Peretti (2002), which has been used to select what assets to include in this study as well as the sample period. They have subsequently been excluded from this analysis as well. Personal sector holdings of national savings have been downloaded from DataStream and the Office for National Savings has contributed the returns on this asset.

In the construction of the user costs, the benchmark rate in Binner, Elger and de Peretti (2002) has been used⁹. All asset quantities have been seasonally adjusted using the X11 routine in Eviews 4.0 using default values and converted to per capita quantities by dividing by a measure of the population size. All conversions between real and nominal variables have been performed using a X11 seasonally adjusted consumer price index (CPI) from the national accounts in the OECD statistical compendium¹⁰. Graphs of the data used in the estimation of the demand system that only contains monetary assets are given in Appendix A.

In the attempts to obtain theoretically consistent estimation results from first stage estimation of the demand system, several different combinations of consumer non-durables, durables, services and leisure were used. This data set is described in Binner, Elger and de Peretti (2002).
IV. EMPIRICAL APPLICATION

The choice of model in this paper has been motivated by the fact that the variables may be non-stationary and it is therefore appropriate to discuss how severe these problems may be in the current data set and also how general these problems are. Various forms of non-stationarity and potential solutions in empirical applications are discussed in Enders (1995). Following Enders, a variable \( y \) observed over time is said to be covariance stationary if

\[
E(y_t) = E(y_{t-s}) = \mu \quad (\mu \text{ is the mean}),
\]

\[
E[(y_t - \mu)^2] = E[(y_{t-s} - \mu)^2] = \sigma^2 \quad (\sigma^2 \text{ is the variance in } y)
\]

and

\[
E[(y_t - \mu)(y_{t-s} - \mu)] = E[(y_{t-s} - \mu)(y_{t-j-s} - \mu)] = \gamma^2 \quad (\gamma^2 \text{ is the covariance}).
\]

Many non-stationary processes can be transformed to (covariance) stationary processes by differencing. Such a process is said to be integrated of order one, I(1). Some processes may also be transformed to stationary processes by removing linear (or possibly quadratic) trends in the data. The most commonly used tests in the literature for non-stationarity are based upon general linear autoregressive models in order to investigate if the process contains a single unit root. These tests make more or less restrictive assumptions about the residuals.

In the more recent time series econometrics literature (see i.e. Johansen (1995)), multivariate stationarity tests are generally performed within a cointegrated vector autoregressive model framework. The basic idea is that individual variables may be nonstationary, but a linear combination of variables may be stationary. If this is the case, these variables are cointegrated. If a variable is stationary, that variable must be a cointegrating vector itself (Johansen (1995)). In the current application, there are by far too few observations available in order to perform the analysis within a cointegration framework. Another problem encountered in this model set-up is that the budget shares are bounded to the (0,1) interval. Hence, they cannot wander away infinitely and they are not per definition nonstationary. Still, they may appear nonstationary in the interval under observation.
A visual inspection of \( \ln(x_t - \ln P_t) \) indicates that this series non-stationary in the current data set and this is supported by a univariate Phillips Perron (1988) unit root test. This test has been chosen since it is robust to heteroscedasticity and autocorrelation of unknown form. The test statistic is \(-1.03\) (using four lags and an intercept) and since this is less than the \(-2.97(-2.62)\) MacKinnon (1991) critical value on the 95(90) percent level, the null of a unit root cannot be rejected. The null can is rejected for all expenditure shares except IBSD, where the test statistic is \(-0.70\). The univariate Phillips Perron tests indicate that all logged user costs are stationary. These results are clearly dependent on the short sample under investigation. Nominal user costs are calculated by multiplying real user costs with a true cost of living index (here approximated by the CPI). In a very large number of papers utilizing cointegration techniques, the price level is found to be integrated of order one or higher (see i.e. Binner and Elger (2002)). It is likely that this should affect the time series properties of the individual user costs.

Recognizing the problems related to testing for nonstationarity using univariate tests, the insufficient number of observations for multivariate tests as well as common findings in empirical studies suggesting that at least a few of the variables should be nonstationary, the demand system was estimated in levels (as defined in (8) using the linearisation in (9)) to look for signs of non-stationarity. One common sign is that the residuals are autocorrelated. Considering the residuals from the demand system that contains monetary assets and using the ordering of the variables in table one, the Durbin Watson test statistics are 1.46, 1.54, 1.18, 0.92, 1.10. Hence the residuals are autocorrelated. Tests for higher order autocorrelation are given in table one for the corresponding differenced system. It is clear that differencing does not remove all problems related to autocorrelation, even though the null of no first order autocorrelation cannot be rejected using a system test.
Demand System Estimation

Since weak separability (approximately) implies two stage budgeting, attempts were made to estimate a first stage demand system that included consumer goods, durables, services and leisure. An aggregate real monetary services index and a dual user cost index was constructed for the monetary assets following Barnett (1980). Various combinations of assets were tested, but these yielded either insignificant or theoretically inconsistent results.

In the second stage of the analysis, a demand system is estimated that only contains monetary assets. In the linearisation of the AI demand system, as suggested by Deaton and Muellbauer (1980a,b), it is common to use prices expressed as price indices (from the national accounts). Deaton and Muellbauer (1980b) motivate the use of the Stone price index (defined in (9)) by the observation that price indices are collinear in most practical situations and that the specific choice of aggregate price index should therefore be unimportant. Formal tests revealed that the estimation results were practically identical regardless if the Stone price index was constructed from the individual user costs or if a dual price index was used (constructed by calculating total expenditure and a real quantity index). In order to further evaluate Deaton and Muellbauer’s proposition, price indices were constructed for each of the individual monetary assets and a Stone price index was constructed based upon these individual indices. The correlation coefficient between this price index and the dual price index defined in (6) above is 0.9988. Hence, the practical effect of which price index that is chosen is negligible. Based on these findings, a dual price index was used. The main motive being that it is the index used in any first stage estimation.

[TABLE 1 HERE]
It can easily be verified that the columns containing $\gamma$ and $\beta$ sum to zero (ignoring round-off errors). It is interesting to note that $R^2$ is much higher than what is common for demand systems estimated in differenced form. It is likely that this is an effect of using very disaggregated data with a large variability in the prices. All $\gamma_{ii}$ coefficients differ significantly from zero as well as some of the $\gamma_{ij}$ coefficients and the majority of the $\beta$ coefficients. Real expenditure is not significant in the IBSD and IBTD equations. All residuals are normal distributed, which is also supported by a multivariate Doornik-Hansen test ($p=0.98$). The univariate Breusch-Godfrey tests reveal that differencing does not remove all problems related to autocorrelation. A systemwise first order Breusch-Godfrey autocorrelation test (see Edgerton, et al (1996, p. 80) shows, however, that the null of no autocorrelation cannot be rejected. The value of the test statistic is 6.75. This statistic is distributed $\chi^2(4)$, which gives a p-value equal to 0.15\footnote{11}. A multivariate test against autoregressive conditional heteroscedasticity based on four lagged residuals (ARCH(4)) test shows that the null of homoscedasticity cannot be rejected, which supports the findings in the univariate tests. The test statistic is 4.34, which in the asymptotic $\chi^2(5)$ distribution gives a p-value equal to 0.50. A Wald system homogeneity test suggest (as expected from the single equation estimation results) that the null of homogeneity cannot be rejected, $\chi^2(4)=1.98(p=0.74)$. A system homogeneity and symmetry test further suggests that this joint hypothesis cannot be rejected, $\chi^2(10)=5.8830(p=0.82)\footnote{12}.$

**Elasticity Estimates**

Short-run uncompensated ($e_x$) and compensated ($e_x^{\sim}$) own- and cross elasticities are given in Table 2 below together with income elasticities ($E_i$) in Table 3 evaluated at mean expenditure shares. The own- and cross price elasticities are calculated using the formulae suggested by Chalfant (1987)\footnote{13}: 
\[
\begin{aligned}
e_{ij} &= \left(\gamma_{ij} - \beta_i s_j\right)/s_i & \quad i \neq j \\
e_{ij} &= \left(\gamma_{ij} - \beta_i s_j\right)/s_i - 1 & \quad i = j
\end{aligned}
\tag{10}
\]

\[
\begin{aligned}
\bar{e}_{ij} &= \gamma_{ij}/s_i + s_j & \quad i \neq j \\
\bar{e}_{ij} &= \gamma_{ij}/s_i + s_j - 1 & \quad i = j
\end{aligned}
\tag{11}
\]

\[E_i = 1 + \beta_i/s_i\tag{12}\]

It is often argued that the budget shares used in the calculations of the elasticities should be replaced with the estimated budget shares (see i.e. the discussion in Edgerton et al (1996, p. 163)). Forecasts of the budget shares indicated (similar to what is commonly found) that the differences were very small and actual budget shares have therefore been used. In order to perform tests if the elasticities differ from zero, the delta method has been used\(^{14}\). In doing so, the mean expenditure share is treated as a fixed coefficient\(^{15}\).

**[TABLE 2 HERE]**

First it should be noted that the elasticities vary over time depending on the fact that the expenditure shares vary over time. The time paths of the uncompensated own-price elasticities are given in Appendix B.

The (significant) estimated uncompensated own-price elasticities are negative (inelastic) or do not differ significantly from zero. Considering the graphs in Appendix B, it is evident that the own-price elasticity for NS is above zero over portions of the sample. It does not, however, differ significantly from zero. The own-price elasticity for NI is (marginally) positive on two occasions in the sample under investigation. For the non-interest bearing assets, one must bear
in mind that this asset is constructed from two different assets (NC and NIBD). Even though the user cost is identical for both variables, this does not necessarily mean that they deliver the same degree of monetary services. It is likely that the services they deliver differ, but this is not reflected in the user cost calculation. It would be desirable to be able to model other asset characteristics into the user cost, but there is no theoretically justifiable method available for this purpose. It is also difficult to evaluate the quality of the data on national savings (NS) since it does not originate from the Bank of England. What appears more problematic is that all compensated own-price elasticities are non-negative. The observed values are, however, small and do not differ from zero. Hence, it cannot be concluded that they violate economic priors.

[TABLE 3 HERE]

The estimated income elasticities are highly significant for a majority of the included assets. A finding that $E_i>1$ implies that this good is a luxury. The hypothesis that IBTD is a luxury good cannot be rejected ($p=0.03$). Any finding suggesting that non-interest bearing assets or bank sight deposits were luxuries would have been controversial, but IBTD is a less liquid asset. The time paths of the income elasticities are given in Appendix C.
V. CONCLUSIONS

By treating monetary assets as durable goods, it is possible to estimate the demand for money using standard consumer demand theory. This allows for further testing of various hypotheses dictated by microeconomic theory. Further establishing the microfoundations for money demand is an important task, since it will lead to a better understanding of how changes in interest rates (operating through the user costs) affect the demand for monetary assets.

The empirical estimation of demand systems is a complex matter. As discussed in the introduction, many economists have proposed using functional forms that are “globally flexible”. These functional forms are highly non-linear in the parameters. The main argument is that when variations in prices are at all large, demand systems that are only locally flexible (such as the AI demand system) perform badly. One fundamental problem in using highly non-linear demand systems is that many time series variables used in empirical estimations are non-stationary. For example, nominal user costs for monetary assets are constructed by multiplying real user costs with a proxy for the true cost of living index. In general, the CPI is used as a proxy for the true cost of living index and this index is frequently found to be I(1) or higher in studies based upon the cointegration framework. Nonstationarity has been dealt with extensively in the macroeconomics literature, but less so in the microeconomics literature. One advantage of using a locally flexible demand system, such as the AI demand system, is that it can be linearised and estimated in differenced form. This is a convenient way to deal with non-stationarity. Formally dealing with nonstationarity is essential to attract attention to the demand systems approach from applied macroeconomists and time series econometricians.
In the current study, an AI demand system is estimated in differenced form using data for the period 1991Q4 to 1998Q4. This sample has been chosen based upon a weak separability study by Binner, Elger and de Peretti (2001). The demand system is estimated in two stages. In the first stage of the analysis, all coefficient estimates are theoretically inconsistent or insignificant. In the second stage, a demand system containing five different monetary assets is estimated. In this system, a majority of the coefficient estimates are significant. The system conforms to homogeneity and symmetry restrictions imposed by microeconomic theory. All significant uncompensated own-price elasticities are negative, and none of the compensated own-price elasticities differ significantly from zero.

These findings provide a strong support for modeling the demand for monetary assets within a standard consumer demand framework. Dealing explicitly with non-stationarity by differencing and using a sample that satisfies the weak separability criterion produces theoretically consistent estimates even when a linearised system is used. Dealing explicitly with non-stationarity in non-linear demand system specifications is a challenging task for future studies. In the ideal world, the demand for monetary assets should be estimated within a nonlinear multivariate cointegration / error correction framework that is firmly based in microeconomic theory.
This research project has been financed by Jan Wallander och Tom Hedelius stiftelse (J98/14). Additional funding has been obtained by the Swedish Royal Academy of Sciences, Crafoordska Stiftelsen, Stiftelsen för Främjande av Ekonomisk Forskning i Lund and the Swedish Royal Academy of Sciences. A number of persons have contributed with valuable comments, in particular David Edgerton at Lund University, Jane Binner at Nottingham Business School, Philippe de Peretti at Sorbonne and Barry Jones at Binghamton University. Lisbeth Funding la Cour at Copenhagen Business School commented on an early version of this paper based on a different data set. This paper has been accepted for presentation at the SMYE conference in Paris, April 2002. An earlier version of this paper was presented to the Money, Investment and Risk conference at Nottingham Trent University, 1999.

1 An overview of various demand system specifications is given in Serletis (2001, Ch. 17).

2 See i.e. Edgerton et al (1996, Chapter 7) for a further discussion and references on demand system estimations and nonstationarity.

3 Regarding bias arising from the use of the Stone price index, see Pashades (1993), Buse (1994) and Alston, Foster and Green (1994).


5 \[ V(m) = V_1(m_1, V_2(m_2)). \] \( m_2 \) contains the two non-interest bearing assets in the system and \( m_1 \) contains the remaining monetary assets. It would have been possible to combine non-interest-bearing assets with other bank deposits, but aggregating the two most liquid assets makes sense economically.

6 In line with these ideas, but from a different point of view, Barnett, Jones and Nesmith (2000) study the demand for money in the US within a linear cointegrated VAR model framework. They find a single stationary relationship (cointegrating vector) that may be interpreted in terms of a money demand relationship. Formal tests suggest that the error correction term contains non-linearities and based upon this finding they argue that stationarity is not sufficient for linearity in linear cointegrated VAR models.

7 Referring to the theory as “monetary aggregation theory” is based upon the title of Barnett and Serletis (2000) book.

8 In first differences, the differences in the expenditure shares add up to zero non-stochastically.

9 This benchmark rate has been calculated as the upper envelope of the returns on all assets included in this study as well as the expected returns on equity, bonds and unit trusts.
This index is equivalent to the retail price index (RPI) monitored by the Bank of England, but differently normalized. The CPI is 1.0 in mid 1995.

Edgerton and Shukur (1999) advocate using the small sample correction suggested by Rao (1973) (see also Edgerton et al (1996, p. 83)). Using Rao’s F-test, the test statistic for the homogeneity test is 0.88. This statistic is distributed F(4,14), which gives a p-value equal to 0.50.

Using Rao’s F-test, the test statistic for the homogeneity test is 0.34. This statistic is distributed F(4,19), which gives a p-value equal to 0.85. For the joint hypothesis of homogeneity and symmetry, the test statistic is 0.40. This statistic is distributed F(10,44.95) giving p=0.94.

For discussions of the validity of these elasticities, see Edgerton (1996, 1997).

See i.e. Greene (2000). These tests have been performed using analyze in TSP, ver. 4.3.

It is also important to make a distinction between total- and within group elasticities. Generally, one is interested in the total elasticities and not the within group elasticities. Total and within group elasticities are identical if the estimated between group own-price elasticities are equal to –1 (see Edgerton (1997)). In the case that the estimated coefficients in stage one of the analysis are insignificant, the uncompensated between group own-price elasticities equal –1 (consider i.e. equation 10 above when \(i=j\)), while the compensated between group elasticities equal \(s^{-1}\).
ABBREVIATIONS

AI: Almost Ideal
AIM: Asymptotically Ideal Model
ARCH: Auto Regressive Conditional Heteroscedasticity
BSD: Building Society Deposits
CPI: Consumer Price Index
IBSD: Interest-Bearing Bank Sight Deposits
IBTD: Interest-Bearing Bank Time Deposits
MSI: Monetary Services Index
NC: Notes and Coins
NI: Non-Interest bearing assets
NIBD: Non Interest Bearing Bank Deposits
NS: National Savings
VAR: Vector Auto Regressive
REFERENCES


Appendix A. Data
Appendix B. Uncompensated Own-Price Elasticities
Appendix C. Income Elasticities
Tables 1-3

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Note: All values in parenthesis are p-values. Bold coefficient estimates differ significantly from zero on the 90 percent level. Norm refers to a Jarque-Bera normality test, HET refers to an ARCH(4) Lagrange multiplier test, which is distributed $F(4,19)$, COR refers to a Breusch-Godfrey autocorrelation test including 4 lags, which is distributed $F(4,14)$. H refers to an equationwise price homogeneity test and is distributed $F(1,22)$. Under the respective nulls, the errors are distributed normal, homoscedastic and not autocorrelated.
Table 2: Uncompensated and Compensated Own and Cross Price Elasticities Evaluated at Mean Expenditure Shares

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Note: Values in parenthesis are p-values from Wald tests for the hypothesis that the elasticities are zero. These test statistics are distributed $\chi^2(1)$.  


Table 3: Income Elasticities Evaluated at Mean Expenditure Shares

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*Note: Values in parenthesis are p-values from Wald tests for the hypothesis that the elasticities are zero.

The test statistic is distributed $\chi^2(1)$. 