Increasing Returns, Input-Output Linkages, and Technological Leapfrogging

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Abstract

Firms agglomerate in one region due to increasing returns, input-output linkages and transportation costs. In the de-industrialised region factor prices are lower and a new technology may be profitable to adopt in that region instead, inducing a change in the technological leadership. This paper shows that the risk of locking in to an old technology is monotonically increasing in the benefits of agglomeration. Greater incompatibility between technologies also increases the risk of rejecting potentially superior manufacturing processes.

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1 Introduction

Economic leadership has shifted among countries throughout the course of history. Maddison (1991, p. 31) notes that "By 1700 Dutch income per head was around 50 per cent higher than that in the nearest rival, the UK, and its economic structure was more advanced." Yet, a century and a half later the UK had surpassed the Netherlands. The British lead, in turn, ended with the rise of the US at the turn of the 19th century (Habakkuk, 1962, Maddison, 1991, and Brezis et al., 1993). Perhaps an even more striking example is the uneven development that took place between Belgium and the Netherlands. According to Mokyr (1976, p. 261) “... it seems likely that during the first half of the nineteenth century the Netherlands and Belgium exchanged positions. While the former was definitely the richer country in terms of GNP per capita in around 1795, it seems very likely that the latter overtook it before the midcentury...” Such changes in economic leadership are not confined to countries. From a historical perspective there are many episodes of economically advanced regions, industries and cities losing their leadership after some technological breakthrough.

This paper analyses whether such a process of catching up and overtaking can be explained by agglomeration economies. An illustrative example is the rise and fall of the Dutch rural region Zaan as a shipbuilding and lumber milling centre. When the Amsterdam sawyers’ guild successfully resisted the introduction of mechanised sawing for decades, the Zaan region with its lower wages (and other competitive advantages such as cheap land and unobstructed access to the wind) attracted the wind-powered lumber mills instead. The ready access to a large and varied supply of lumber inventories induced a relocation of the shipbuilding industry, and the Zaan region rose rapidly to dominance in Dutch shipbuilding after 1600. The number of wharves increased from 13 in 1608 to 60-65 around 1670. In addition, out of a total of 86 sawing

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1 For example, Ames and Rosenberg (1963, p. 14) assert that “It is certainly a fact that the countries whose industries have grown fastest in the past one hundred years are not those which grew most rapidly in the preceding century. The leading industrial countries of the middle ages – Brabant, Lombardy, Venetia – have never regained their former position in the world, any more than Egypt has regained rule over the grain trade.”

2 This example is from Vries and Woude (1997, pp. 296-303).
windmills in all Holland in 1630, 53 were located in the region. A century later it contained 256 of Holland’s 448 sawing windmills. At its zenith between 1680 and 1730, the Zaan region seemed to hold an unchallengeable position as a shipbuilding and lumber milling centre. However, toward the end of the seventeenth century new shipbuilding techniques, employed by the English and French, emphasised the use of iron, while the Dutch, due _inter alia_ to the strong links between shipbuilding, lumber milling and timber trade, continued to employ methods based on wood. The loss of foreign demand for ships gradually eroded the Zaan region’s position as a shipbuilding centre, with only 2 or 3 wharves in 1792; by 1816 the industry had ceased to exist. The collapse of the shipbuilding industry caused demand for sawn lumber to decline even further, and the mechanised sawing industry in the Zaan region also contracted.

Explaining the geographical concentration of industry, the woodworking industries of Zaan being an example, is at the very heart of the _new economic geography_. According to this literature, the main benefit to firms and consumers in an industrial centre is better access to more and cheaper varieties of inputs and goods. The drawbacks of locating in such a centre, besides the standard centrifugal forces such as increased competition on product and factor markets, have not been researched as extensively as the benefits. However, Venables (1996b) argues that there may be a risk of technological lock-in as firms hold on to an old technology instead of adopting a new and more productive one. The informal argument is that firms may be unwilling to give up the agglomeration benefits associated with the existing technology. There is also a risk that the new technology may be adopted in a less industrialised region featuring lower factor prices, and hence that the new industry will be

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3 According to Vries and Woude (1997, p. 301), the concentration of the woodworking industries in this area was so high that "No other place possessed anything remotely comparable to this industrial concentration."

4 The field has grown rapidly since the seminal contributions by Krugman (1991), Krugman and Venables (1995) and Venables (1996a). Ottaviano and Puga (1998) offer an easily accessible survey of the early models, whereas a more formal exposition is provided in Fujita et al. (1999). A second generation of models can be found in Baldwin et al. (2003). For some critical assessments of the _new economic geography_, see Martin (1999) and Neary (2001).
established elsewhere, as shown by Amiti (2001).

In this paper we introduce a model examining the relationship between technological lock-in and agglomeration benefits. Industry agglomerates in one region due to backward and forward linkages among firms. We show that these linkages may induce the firms in the industrial centre to hold on to an old technology, as illustrated above by the woodworking industries of Zaan. We also analyse how the degree of incompatibility between technologies interacts with agglomeration economies in the adoption process. To investigate these issues we extend the model developed by Krugman and Venables (1995), and allow for technological progress by assuming an exogenous supply of new technologies to firms in the differentiated sector. Starting from a core-periphery equilibrium, we analyse if and where new technology is adopted. We show that the possibility of firms holding on to old technologies increases the greater the benefits of agglomeration and the less compatible technologies are. However, the new goods may be less expensive if produced in the periphery instead, because the wage is lower there. Should it be profitable for firms to adopt the new technology in the peripheral region, the industrial structure in the two regions will change, and an agglomeration of firms using the new technology will be established in the former periphery, completely reversing the old pattern of specialisation.

The rest of this paper is organised as follows. Section 2 reviews previous research, and points out how this paper differs from it. In section 3 the model is introduced, section 4 analyses the conditions under which technological lock-in and leapfrogging are possible and, finally, section 5 offers some concluding remarks.

2 Previous Research

The factors underlying cycles of leadership between countries, regions, and industries are, of course, a complex mix of differences in institutional and social structures, factor endowments and technological progress. Yet one interesting idea, going back at least to Veblen (1915), is that there is “a penalty for taking the lead.” Technical progress leaves the pioneer with obsolete equipment. Followers can avoid the mistakes (and the costs incurred by them) made by
the leader, bypass old technologies and adopt more modern methods of production, and develop and adopt uniform standards relatively faster. In the end they can catch up with and even overtake the leading country.\textsuperscript{5}

More recently, Brezis et al. (1993) argue that leading countries' failure to adopt new technologies may be caused by externalities associated with existing technology. They develop a Ricardian model of trade and growth where shifting leadership roles are due to country-specific learning-by-doing effects. When technological progress constitutes major shifts in the current technological paradigm (revolutions), it renders the existing knowledge useless. The leading nation may then continue to use an old technology which, given the experience the leader has accumulated using it, is more productive than a new one with which it has no experience. In the lagging nation with less experience and lower wages, however, adoption of the new technology may be profitable. If it is, production using the new technology will be set up in that country and at some point it will surpass the more advanced nation.\textsuperscript{6}

Desmet (2002) extends the results in Brezis et al. (1993) by allowing inter-technological spillovers. He shows that if such spillovers are strong, then the leading country may adopt the new technology despite its higher wage level. If they are weak we may observe a process of endogenous leapfrogging. Amiti (2001) shows that agglomeration economies can also cause regions to leapfrog in technology. Vertical linkages between an upstream Cournot oligopoly and a perfectly competitive downstream industry give rise to an industrial centre and a less industrialised periphery, creating a regional difference in wages. Because different technologies are assumed to be completely incompatible, a firm considering using the new technology receives no agglomeration benefits from locating in a region with many firms using the old technology. The only difference between the regions when choosing a location is hence the wage rate, and the new technology is therefore more expensive to adopt in the industrial centre than in the periphery. A possible equilibrium is the peripheral region

\textsuperscript{5}For a more elaborated discussion of the "handicap of the early start" see Jervis (1947), Frankel (1955), Ames and Rosenberg (1963), Kindleberger (1961 and 1964) and the references therein.

\textsuperscript{6}Brezis and Krugman (1997) develop a similar model to explain rotating leadership between cities. Arthur (1989) contains an analysis of technology choice when adoption is characterised \textit{inter alia} by increasing returns.
taking over the technological leadership.

The main contribution of our paper is that the basic mechanism causing technological leapfrogging has micro-foundations and is mediated through markets (in contrast to Brezis et al., 1993, and Desmet, 2002), and that we allow for compatibility between old and new technologies (in contrast to Amiti, 2001). The latter enables us to distinguish between technological revolutions, introducing a discontinuity with the past, and non-drastic innovations, weakening (but not completely destroying) linkages across technologies. We can also directly relate the risk of technological lock-in to the standard benefits of agglomeration highlighted in the new economic geography. This yields a richer analysis of how agglomeration economies and the nature of technological progress interact in the adoption process of new technologies.

3 The Economic Model

The world consists of two countries or regions, North and South, that are identical regarding factor endowments, preferences and technology. We use an asterisk to denote South’s variables. Each region is endowed with one unit of labour. Labour is interregionally immobile, but can move between sectors within a region. There are two industrial sectors. One, the $A$ sector, produces a homogeneous good with constant returns to scale and its firms are price takers. The other, the $M$ sector, is a monopolistically competitive industry producing differentiated goods under increasing returns to scale. We extend the Krugman and Venables (1995) framework by allowing for the existence of future new technologies in the differentiated sector. The supply of these new technologies is assumed to be exogenous. We use the subscript $i$ to denote variables relating to the existing technology, whereas $i + 1$ is used to denote variables associated with the new technology.

Homogeneous goods are traded freely, while interregional transportation of differentiated products incurs iceberg costs. For every unit of a variety arriving at a destination, $t > 1$ units have to be shipped from the other region. Consumer preferences are represented by
where \( A \) is consumption of the homogeneous good, \( M \) is consumption of the composite of all the differentiated varieties, and \( n_\psi \) is the mass of firms in North using technology \( \psi = i, i + 1 \). The elasticity of substitution between any pair of the differentiated products is given by \( \sigma > 1 \); it is also the price elasticity of demand. Differentiated goods produced with the new technology enter the utility function symmetrically and are added to the consumption of old goods. North’s consumer price index of the manufacturing aggregate \( M \), given the subutility function in (1), is

\[
G^c = \left[ n_i p_i^{1-\sigma} + n^*_i (tp^*_i)^{1-\sigma} + n_{i+1} p_{i+1}^{1-\sigma} + n^*_{i+1} (tp^*_i)^{1-\sigma} \right]^{\frac{1}{1-\sigma}},
\]

where \( p_\psi \) is the price charged for each variety produced with technology \( \psi \) in North.

The production function of the \( A \) sector is \( X^A = 1 - \lambda_\psi \), where \( \lambda_\psi \) denotes the share of labour working in the \( M \) sector using technology \( \psi \). Choosing the homogeneous good as \textit{numéraire} imposes the following restriction on the manufacturing wage rate, \( \omega_\psi \geq 1 \), due to the free mobility of labour across sectors.

The production function is Cobb-Douglas in labour and the manufacturing aggregate \( M \), where the latter’s share equals \( \alpha \). A crucial difference between our model and Krugman and Venables (1995) is that the intermediate input aggregate need \textit{not} be the same CES aggregate of \( M \) sector varieties as the consumption good. This has to do with how we choose to think of technological progress. We assume that the goods produced with the new technology are \textit{completely useless} in the production of old goods. This is simply to say that the arrival of new technology cuts off the demand linkage for the adopting firms; they lose intermediate demand from the firms producing old goods. We also assume that the emergence of new technology weakens the cost linkage: the firms considering adoption are able to use the old goods as inputs, although not to the same extent as firms using the old technology. This has
important implications for the CES aggregates of intermediate inputs. For firms continuing using the old technology, the input aggregate differs from the consumer aggregate as new goods are not added to old ones on the production side (new goods are assumed to be useless in the production of old goods). For the adopting firms, however, the new goods are added to the old ones (this follows from the assumption that adopting firms can use old goods as inputs), but their aggregate still differs from consumers’ as will be clear below. Specifically, the producer price index for firms using the old technology is

\[ G_p^i = \left[ n_ip_i^{1-\sigma} + n_i^* (tp_i^*)^{1-\sigma} \right]^{\frac{1}{1-\sigma}}, \]  

(3)

whereas the producer price index facing the adopting firms is\(^7\)

\[ G_{p+1}^i = \left[ \theta \left( n_ip_i^{1-\sigma} + n_i^* (tp_i^*)^{1-\sigma} \right) + n_{i+1}p_{i+1}^{1-\sigma} + n_{i+1}^* (tp_{i+1}^*)^{1-\sigma} \right]^{\frac{1}{1-\sigma}}, \quad \theta \in [0, 1]. \]  

(4)

The parameter \( \theta \) measures how compatible the new technology is with the old one. If this “compatibility parameter” is unity, then the old goods are perfectly usable as intermediates in the production of new goods (and \( G_p^{i+1} \) will be identical to \( G^c \)); if it is zero they are useless. The bigger the break with the past the new technology constitutes, the lower is \( \theta \). We can thus use \( \theta \) to distinguish between major technological breakthroughs and smaller improvements within the existing technological paradigm. This extends the analysis in Amiti (2001), where technologies are assumed to be completely incompatible.\(^8\)

Production of a given amount of output using technology \( \psi \left( q_M^{\psi} \right) \) requires a fixed cost of production \( (\beta_{\psi} F) \) and a constant marginal cost \( (\beta_{\psi} C_M) \) in terms of the composite input. The total cost per firm using production process \( \psi \) then is

\[^7\text{I am grateful to Frédéric Robert-Nicoud for this suggestion.}\]

\[^8\text{She discusses briefly how allowing (perfect and imperfect) compatibility would reduce the likelihood of leapfrogging, but the paper contains no explicit analysis of the issue.}\]
\[ TC_\psi = \omega_\psi^{1-\alpha} (G_\psi^0)^\alpha \beta_\psi (F + c^M q^M_\psi), \]

\[ \beta_\psi = \begin{cases} 1 & \text{if } \psi = i \\ \in (0, 1) & \text{if } \psi = i + 1 \end{cases}. \]  

(5)

The parameter \( \beta \) captures the fact that a new technology is more productive than an old one: any given output level requires less of the composite input and is ceteris paribus produced at lower cost. From equations (3) and (5) we see that adding a firm to North benefits other firms in that region since it lowers the price index and thereby reduces each firm’s costs. This is the forward linkage causing agglomeration and it differs somewhat from the forward linkage in Amiti (2001). In her framework downstream firms benefit as the input price is driven down by entry into the upstream Cournot sector (there is no trade in inputs). Here, firms also benefit from avoiding trade costs as fewer of their input goods have to be imported.

Profit-maximising behaviour by firms ensures that marginal revenue equals marginal cost:

\[ p_\psi \left( 1 - \frac{1}{\sigma} \right) = \omega_\psi^{1-\alpha} (G_\psi^0)^\alpha \beta_\psi c^M, \]

where \( c^M = 1 - \frac{1}{\sigma} \) by normalisation. Total expenditure on differentiated products in North, \( E_\psi \), is the sum of consumer demand and firms’ expenditure on intermediates:

\[ E_\psi = \mu Y_\psi + \alpha n_\psi p_\psi q^e_\psi, \]

where \( Y_\psi = \omega_\psi \lambda_\psi + 1 - \lambda_\psi \) is the income in North, \( \mu \) is the share of income spent on differentiated products, \( \alpha \) is the intermediate share in production and \( q^e_\psi \) is equilibrium output per firm. Total demand for an individual variety is

\[ d_\psi = \frac{p^{-\sigma}_{\omega_\psi} E_\psi}{G_\psi^{1-\sigma}} + \frac{p^{-\sigma}_{\omega_\psi} q^{1-\sigma}_{\star} E^*_{\psi}}{(G^*_{\star})^{1-\sigma}}. \]

(8)

The first term is domestic demand and the second is Southern demand, including the goods that are used up in transit. Due to free entry and exit, profits

\footnote{Note that we have to distinguish between the price indices facing consumers and producers in the denominators once new technology arrives. We elaborate on this point in section 4.}
are zero in equilibrium: 
\[ p_\psi q_\psi^e = \omega_\psi^{1-\alpha} \left( G_\psi^p \right)^\alpha \beta_\psi \left( F + c^M q_\psi^e \right). \]
Solving for the supply per firm and using equation (6) yields \( q_\psi^e = F\sigma \). We normalise output per firm by choosing \( F = \frac{1}{\sigma} \). In equilibrium the market for differentiated goods clears, \( q_\psi^e = d_\psi \):

\[
1 = p_\psi^{-\sigma} \left( \frac{E_\psi}{G_\psi^{1-\sigma}} + \frac{t^{1-\sigma} E_*^\psi}{(G_*^\psi)^{1-\sigma}} \right)^\lambda. \tag{9}
\]

If the number of firms located in North increases, then the market for intermediates will be larger, and so will total expenditure on differentiated goods as shown by the second term in equation (7). This raises demand and profits per firm via equations (8) and (9). This **backward linkage** is the second agglomeration-generating force in the model.

Finally, the wages paid by Northern firms using technology \( \psi \) equal the labour cost share of total costs, which in equilibrium amount to total revenue:

\[
\omega_\psi \lambda_\psi = (1 - \alpha) n_\psi p_\psi q_\psi^e. \tag{10}
\]

This completes the description of the model. Our starting point is a core-periphery equilibrium, by which we mean that all the \( M \) sector firms using the current technology are located in North. That is, we assume that economic integration has led to a deindustrialisation of South, which hosts only the \( A \) sector.\(^{10}\) We will refer to North as the core and South as the periphery. Solving the model\(^ {11}\) gives the following expressions for the equilibrium mass of firms and the price of each variety in North when the old technology is used:

\[
n_i = \left( \frac{\mu \omega_i + \mu \omega^*}{(1 - \alpha) \omega_i} \right)^{(1-\sigma)(1-\alpha)} (1 - \sigma + \sigma \alpha) ; \quad p_i = \omega_i \left( \frac{\mu \omega_i + \mu \omega^*}{(1 - \alpha) \omega_i} \right)^{\frac{\sigma}{1 - \sigma + \sigma \alpha}}. \tag{11}
\]

Inserting these\(^ {12}\) in equation (10), using \( \lambda_i = 1 \) and solving for the Northern

\(^{10}\)Throughout the paper we choose values of \( \alpha, \mu, \sigma \) and \( t \) ensuring that, in the absence of technological change, the initial core-periphery equilibrium is stable; see Appendix A.1.

\(^{11}\)We use \( n_1^* = 0, \lambda_i = 1, Y_i = \omega_i, \lambda_i^* = 0 \) and \( Y_i^* = \omega^* \) in equations (2), (3), (6), (7), (9), and their Southern counterparts to solve for \( n_i \) and \( p_i \).

\(^{12}\)Note that the factor \( 1 - \sigma + \sigma \alpha \) in the exponents in (11) is assumed to be negative. This is a standard assumption in this model known as the no-black-hole condition (see Fujita et al. 1999, ch. 14). If it is not met, then agglomeration forces are so strong that a core-periphery equilibrium emerges irrespective of the level of trade costs.
wage yields $\omega_i = \frac{\mu}{1 - \mu}$. The wages in the initial core-periphery equilibrium are thus $(\omega_i, \omega^*) = \left( \frac{\mu}{1 - \mu}, 1 \right)$. We can have either nominal factor price equalisation ($\mu \leq 0.5$; if the inequality is strict the wage is determined by North’s $A$ sector and equals unity) or a factor price differential ($\mu > 0.5$). We will focus on the case of a nominal wage differential. We now allow for a new technology to be available to firms in the $M$ sector. Our task is to analyse whether it will be adopted or not and, if it is adopted, in which region.

4 Arrival of New Technology

4.1 Adoption in the Periphery

In this section we study the behaviour of an atomistic firm considering adoption of the new technology in the periphery (the case of adoption in the core is analysed in section 4.2). The first potential adopter has to rely on old intermediate goods. The trade-offs involved in the adoption process are the following:

1. The adopting firm loses all the intermediate demand from firms still using the old technology
2. It has to pay the trade costs of its intermediate inputs
3. The old goods are not perfectly usable as inputs, which further fuels the rise in costs via the producer price index
4. The new technology is more productive than the old one
5. The nominal wage is lower in the periphery

Points 1-3 are the negative effects of adoption and have to be weighed against the positive ones, i.e. 4 and 5. The effects 1, 3 and 4 are new compared to the original model in Krugman and Venables (1995); 2 and 3 are new compared to Amiti (2001). The second effect is not present in her model as there is no trade in intermediate goods; the third is absent as she only covers the case of completely incompatible technologies. In order to analyse how all these effects of adoption interact, we look at the maximum wage the adopting firm can offer to attract workers from the periphery’s $A$ sector and still break even. Writing

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13This is the standard approach when analysing stability; see Fujita et al. (1999, ch. 14).
out the periphery’s counterpart to equation (9) for the new technology, and using the first-order condition from equation (6) yield

$$\omega^*_{i+1} = \left( G^p_{i+1} \right)^{-\frac{1}{1-\sigma}} \left( \beta_{i+1} \right)^{\frac{1}{1-\sigma}}$$

$$\times \left[ \frac{\mu \left( \omega^* \lambda^*_{i+1} + 1 - \lambda^*_{i+1} \right)}{(G^c)^{1-\sigma}} + \frac{t^{1-\sigma} \mu (\omega_i \lambda_i + 1 - \lambda_i)}{(G^c)^{1-\sigma}} \right]^{\frac{1}{1-\sigma}}. \quad (12)$$

The adopting firm faces no intermediate demand from the firms using the old technology in the core, so each country’s total expenditure consists of consumer expenditure only. The firms within square brackets is consumer demand in South; the second one is consumer demand from North. Next, we need to derive the various price indices appearing in (12). Since the firm is atomistic we can set $n^*_{i+1} = 0$ (in addition to $n^*_i = n_{i+1} = 0$) in all price indices. From equations (2), (3) and (4) we then have $G^c = \left( n_i p_i^{1-\sigma} \right)^{\frac{1}{1-\sigma}}$ ($= G^p_i$), $G^c = t G^c$ and $G^p_{i+1} = \theta^{\frac{1}{1-\sigma}} t G^c$. Inserting $n_i = \frac{\omega_i \lambda_i}{(1-\alpha) p_i}$ from (10) and the first-order condition from (6) in the expression for $G^c$, and solving yield

$$G^c = \omega_i \left( \frac{\lambda_i}{1 - \alpha} \right)^{\frac{1}{1-\sigma+\sigma \alpha}} (= G^p_i); \quad G^p_{i+1} = \theta^{\frac{1}{1-\sigma}} t \omega_i \left( \frac{\lambda_i}{1 - \alpha} \right)^{\frac{1}{1-\sigma+\sigma \alpha}}. \quad (13)$$

Inserting the required expressions for the price indices in (12), and using the fact that $\lambda_i = 1$, $\lambda^*_{i+1} = 0$, $\omega_i = \frac{\mu}{1-\mu}$ and $\omega^* = 1$, we arrive at the following wage equation:

$$\omega^*_{i+1} = \beta_{i+1}^\frac{1}{1-\sigma} \theta^{\frac{1}{1-\sigma+\sigma \alpha}} t \frac{\mu}{(1-\mu)}$$

$$\times \left[ t^{\sigma-1} (1 - \mu) (1 - \alpha) + t^{1-\sigma} \mu (1 - \alpha) \right]^{\frac{1}{\sigma(1-\alpha)}}. \quad (14)$$

This is the wage a deviating firm can afford to pay workers currently employed in the $A$ sector and still break even. If it is less than unity, the firm cannot attract any workers to the $M$ sector and production of new goods is not possible. The wage equation in (14) features some important differences compared to the original one in Fujita et al. (1999) (shown in Appendix A.1).

First, in the present setting the deviating firm loses intermediate demand from the core, since firms using the old technology cannot use new goods as
inputs in production. Comparing the wage equation above to the original one, we see that $\alpha$ is missing in the factor associated with $t^{1-\sigma}$. This lowers the wage the firm can offer. The second major difference is that the wage equation contains two new parameters, $\beta_{i+1}$ (capturing the new technology’s higher productivity) and $\theta$ (capturing how useful the old goods are in the production of new goods). Determining these parameters’ effects on the wage the adopting firm can offer is straightforward. Using (14) we have $\frac{\partial \omega_{i+1}^*}{\partial \beta_{i+1}} < 0$ and $\frac{\partial \omega_{i+1}^*}{\partial \theta} > 0$, since $0 < \alpha < 1$ and $\sigma > 1$. The intuition is clear. The more productive the new technology and/or the more useful old goods are in the new production process, the higher the wage the firm can afford will be and the more likely it is that the new technology will be adopted.\footnote{This is also true if adoption is considered in the core, as we will see in section 4.2.} Figure 1 is a plot of the wage equation in (14) against $t$.\footnote{The parameter values used for all figures in the paper can be found in Appendix A.2.} It also contains a plot of the initial equilibrium’s wage equation to show that the arrival of new technology destabilises the (otherwise stable) original core-periphery equilibrium.

**Figure 1. The arrival of new technology and stability**

The horizontal line depicts the wage that \textit{has} to be offered if the firm is to attract workers from South’s $A$ sector; it equals unity. The curve $O$ is the wage

\begin{figure}[h]
\centering
\includegraphics[width=0.5\textwidth]{figure1}
\caption{The arrival of new technology and stability}
\end{figure}
equation in the original equilibrium (i.e. before the arrival of new technology). We see that if \( t \) is, say, 2.4, then full agglomeration in North is sustainable as the wage the firm can offer at that level of trade costs is less than unity; it is not profitable to relocate to South with the old technology. The curve \( N1 \) is given by (14) and shows the wage the firm considering adoption actually can afford to pay if it relocates using the new technology (with \( \theta \) set to unity to isolate the effect of a higher productivity). It is clear that the offered wage at \( t = 2.4 \) is above unity. The agglomeration is not sustainable after the arrival of the new technology and it starts to unravel. The curve \( N2 \) is drawn for the same productivity parameter as \( N1 \), but with a lower \( \theta \): old goods are less usable as inputs for the firm. We see that for a low enough \( \theta \) it will not be profitable to deviate with the new technology despite it being more productive. From (14) it is clear that the wage the firm can afford tends to zero as \( \theta \) tends to zero. In contrast to Amiti (2001), if the arrival of new technology introduces a complete break with the old one, then it will never be adopted. The reason is simply that if the first adopting firm has to rely only on its own output as intermediates, then the producer price index tends to infinity, and hence so do marginal cost and the price set by the firm. With an infinite price, demand tends to zero and the firm is not able to break even.

What is the end result of firms deviating to South with the new technology? Will the old technology be driven out by the new one or will it still be profitable to produce old goods in the core? To answer these questions we need to resort to numerical solutions, which we turn to next.

4.1.1 Technology Choice and Industrial Structure

We need the core’s wage equation associated with the old technology, since it shows what the firms there can afford to pay the workers in North’s \( M \) sector, given that there are firms using the new technology in South. Firms in the core face demand from four sources: consumer demand at home and from abroad; intermediate demand at home and from abroad. The old technology’s wage equation then is
\[
\omega_i = (G^p_i)^{1-\sigma} \left[ \frac{\mu Y_i}{(G^c)^{1-\sigma}} + \frac{\alpha \omega_i \lambda_i}{(1-\alpha) (G^p_i)^{1-\sigma}} \right. \\
+ \frac{t^{1-\sigma} \mu Y^*_i \omega^*_{i+1}}{(G^c)^{1-\sigma}} + \frac{t^{1-\sigma} \alpha \omega^*_i \lambda^*_i}{(1-\alpha) (G^p^*_i)^{1-\sigma}} \left. \right]^{\frac{1}{1-\sigma}}.
\]

(15)

The wage equation for the periphery now takes the form:

\[
\omega^*_i = (\beta^*_{i+1}) \frac{1}{1-\sigma} (G^{p*}_{i+1})^{\frac{1}{1-\sigma}} \left[ \frac{\mu Y^*_i}{(G^c)^{1-\sigma}} + \frac{\alpha \omega^*_{i+1} \lambda^*_{i+1}}{(1-\alpha) (G^{p*}_{i+1})^{1-\sigma}} + \frac{t^{1-\sigma} \mu Y_i}{(G^c)^{1-\sigma}} \right]^{\frac{1}{1-\sigma}}.
\]

(16)

In this section a significant mass of firms adopts the new technology so \(G^c\), \(G^{c*}\) and \(G^{p*}\) have to be derived with this in mind (\(G^p_i\) is the same as in equation (13)). As before, we can set \(n^*_i = n_{i+1} = 0\) in all the price indices. Using (10) and the first-order condition for each technology then yields the following consumer price indices

\[
(G^c)^{1-\sigma} = \omega^*_1 \left( \frac{\lambda_i}{1-\alpha} \right)^{1-\sigma} + t^{1-\sigma} (\omega^*_{i+1})^{1-\sigma+\sigma} \frac{\lambda^*_{i+1}}{(1-\alpha) \beta^*_{i+1} (G^{p*}_{i+1})^{-\sigma}};
\]

(17)

\[
(G^{c*})^{1-\sigma} = t^{1-\sigma} \omega^*_1 \left( \frac{\lambda_i}{1-\alpha} \right)^{1-\sigma+\sigma} + (\omega^*_{i+1})^{1-\sigma+\sigma} \frac{\lambda^*_{i+1}}{(1-\alpha) \beta^*_{i+1} (G^{p*}_{i+1})^{-\sigma}};
\]

(18)

and South’s producer price index, from (4), as

\[
(G^{p*}_{i+1})^{1-\sigma} = \theta t^{1-\sigma} \omega^*_1 \left( \frac{\lambda_i}{1-\alpha} \right)^{1-\sigma+\sigma} \\
+ (\omega^*_{i+1})^{1-\sigma+\sigma} \frac{\lambda^*_{i+1}}{(1-\alpha) \beta^*_{i+1} (G^{p*}_{i+1})^{-\sigma}}.
\]

(19)

We now use equations (15), (16), (13) for \(G^p_i\), and (17)-(19) to trace the effects of adoption in the periphery on the wages that the firms can afford in each region and on the industrial structure. The results are reported in Figures 2A
and 2B below; the only difference between the two is that the new technology is more productive in Figure 2B.

**Figures 2A and 2B. Adoption in the periphery**

On the x-axis we have the core’s share of labour employed in the production of differentiated goods using the old technology; the y-axis is the periphery’s share of labour in the production of differentiated goods using the new technology. The curve $WN = 1$ gives the combinations of $\lambda_i$ and $\lambda^*_i+1$ for which the core’s wage is unity, whereas the curve $WS = 1$ shows the combinations of $\lambda_i$ and $\lambda^*_i+1$ for which the periphery’s wage is unity. To the right of $WS = 1$ the periphery’s wage is less than unity, to the left it is greater than unity. As labour in a country moves between sectors in response to differences in nominal wages, the vertical arrows show the evolution of $\lambda^*_i+1$. For North the wage is above unity below the curve $WN = 1$; above it the wage is below unity and the horizontal arrows show how $\lambda_i$ evolves.

There are two qualitatively different outcomes displayed in the figures. The first is the possibility of co-existing technologies, which is illustrated in Figure 2A. Starting at the original equilibrium $(\lambda_i, \lambda^*_i+1) = (1, 0)$, we see that an infinitely small increase in $\lambda^*_i+1$ will be associated with a wage greater than unity in South ($\omega^*_i+1 > 1$ since we are below $WS = 1$). Hence $\lambda^*_i+1$ will start to increase. At the same time, we are below $WN = 1$ so the actual wage in North
is greater than unity ($\omega_i > 1$). As long as this is true the whole population in North will continue to work in the $M$ sector ($\lambda_i = 1$). Once we reach $S$, however, the evolution of South’s share of labour employed in manufacturing will come to a halt. Some firms will thus deviate to South using the new technology, but it is not productive enough to drive out the old one.

The difference in Figure 2B is that the new technology is more productive (translating into a lower $\beta_{i+1}$), shifting the $WS = 1$ curve up and the $WN = 1$ curve down. Again, starting from $(\lambda_i, \lambda^*_{i+1}) = (1, 0)$ South’s share of labour in the $M$ sector will start to increase. Once we pass the curve $WN = 1$ the wage in North falls below unity and $\lambda_i$ starts to decrease. However, $\lambda^*_{i+1}$ continues to increase since we are still below $WS = 1$. The evolution of the two countries’ manufacturing labour shares will only come to a halt when we reach $(\lambda_i, \lambda^*_{i+1}) = (0, 1)$. That is, the old technology is completely driven out by the new one and we have a reversed pattern of specialisation, where the former periphery has become the new core.\footnote{Note that the symmetric equilibrium is not attained as the agglomeration unravels (as it is in the original model). The reasons are that i) the firms in South use the new technology while the firms in North use the old one, and ii) intermediate demand across technologies is not symmetric.}

Three observations can be made in relation to Figures 2A and 2B.

Co-Existing Technologies vs Leapfrogging First, it is often noted that old and new technologies co-exist for quite some time in fierce competition. Furthermore, new technologies and innovations are initially crude (relative to their full potential) and undergo gradual improvements, which drastically improves their productivity over time. If this is the case, a complete elimination of the old technology is more likely in the present model. That is, if a new and initially crude technology matures and gains further in productivity, then we would eventually move from the case displayed in Figure 2A to the one in Figure 2B. An illustrative example is perhaps the contest between water power and steam power. Even though Watt patented his improvements of the Newcomen steam engine in 1769, it was only in the latter half of the 19th century that steam overtook water as the major power source in US manufacturing (Rosenberg and Trajtenberg, 2004) and in the UK (Crafts, 2004). There were many reasons for this development, but Rosenberg and Trajtenberg (2004)
argue that one important factor in the US was the emergence of the Corliss steam engine. Being more fuel effective and exhibiting superior performance, it was 30-50 percent cheaper in operation than other steam engines and it "greatly contributed to tipping the balance in favor of steam" (Rosenberg and Trajtenberg, 2004, p. 62). Similarly, advances in the design of steam engines in the UK greatly improved their productivity and reduced costs, giving steam the upper hand over water power there (Crafts, 2004).

**Industry Location: The New Pattern of Specialisation**  
Next, we can solve for the long-run equilibrium and get the expressions for the new mass of firms and the price of each variety as (technology subscripts dropped for brevity)

\[ n^* = \beta \frac{\mu T - \frac{1}{2} \sigma}{1 - \sigma + \alpha} \left( \frac{\mu \omega^* + \mu \omega}{(1 - \alpha) \omega^*} \right)^{(1 - \sigma) (1 - \alpha)} \]

\[ p^* = \beta \frac{\mu T - \frac{1}{2} \sigma}{1 - \sigma + \alpha} \omega^* \left( \frac{\mu \omega^* + \mu \omega}{(1 - \alpha) \omega^*} \right)^{(1 - \sigma) (1 - \alpha)} \]

Inserting these into South's equivalent of equation (10) and solving for the wage rate yields

\[ \omega_{i+1}^* = \frac{\mu}{1 - \mu}. \]

The final outcome then, if the new technology is adopted in South and is productive enough, is that North hosts only the \( A \) sector and South only the \( M \) sector. It can be shown that the wage equation for this core-periphery equilibrium is the same as for the initial equilibrium (i.e. before the arrival of new technology; see Appendix A.1). The new core-periphery equilibrium is therefore stable (since \( \alpha, \mu \) and \( \sigma \) are assumed to be the same across technologies).

**Welfare Effects of Leapfrogging**  
Finally, a comparison of (11) and (20) reveals that there are more varieties and lower prices with the new technology in equilibrium compared to the old one (since \( \beta < 1 \) and \( 1 - \sigma + \sigma \alpha < 0 \)). Hence the new consumer price index, \( G_c^* = \beta \frac{\mu T - \frac{1}{2} \sigma}{1 - \sigma + \alpha} \omega_{i+1}^* \left( \frac{1}{1 - \mu} \right)^{1 - \sigma + \alpha} \), is also lower. All workers in South now have a higher wage and a lower price index, so welfare there is clearly higher than before. Is it possible that North is better off despite having lost its industrial centre? Comparing real wages when it was the core, \( k \omega_1 (G_c)^{-\mu} \), where \( k \equiv \mu (1 - \mu)^{(1 - \mu)} \) and \( G_c \) is as in (13) with \( \lambda_i \) set to unity, to real wages now, \( k \omega (tG_c^*)^{-\mu} \), the answer is yes provided that \( \frac{\omega}{\omega_1} \beta \frac{\mu}{1 - \mu} > 1 \). For North there are two negative aspects and a positive
one of the change in industrial structure. The first factor captures the fact that North now has a lower nominal wage ($\omega < \omega_i$), lowering real wages. The second represents the fact that it now has to import all differentiated goods from South, and pay transport costs. This negative aspect is more important the greater the income share spent on differentiated products ($\mu$). The third factor captures the fact that the new consumer price index *per se* is lower, raising real wages. This effect is also more important the greater the share of income spent on the $M$ sector’s goods, as the consumer price index then has a greater weight in the cost-of-living index. If the new technology is productive enough, then the positive effect will dominate the two negative ones, and North will be better off despite having lost the $M$ sector to South.

4.2 Status Quo or Leapfrogging?

We have so far simply ignored the possibility of the new technology being adopted in the core instead of in the periphery. Since this cannot be ruled out *a priori*, we investigate in this section the exact circumstances under which the new technology is profitable to adopt in the core or in the periphery. We also show analytically that greater benefits of agglomeration increase the risk of locking in to old technology no matter where adoption is considered. To analyse these issues we need the core’s wage equation associated with the new technology. Following the same procedure as for the periphery we have:

$$\omega = \beta \theta^{\frac{1}{1+\alpha}} \mu (1-\sigma) \frac{1}{\sigma(1-\alpha)} (1 - \alpha)^{\frac{1}{\sigma(1-\alpha)}} \mu. \quad (21)$$

The productivity and the compatibility parameters affect the wage in the same way as for a firm considering adoption in the periphery. However, there are two major differences for a firm considering setting up production in the core compared to the periphery. The first is that it has to attract workers currently employed in the $M$ sector, instead of the $A$ sector. Those workers already receive a wage equal to $\frac{\mu}{1-\mu} > 1$, which is what the wage the adopting firm offers should be compared to. A potential adopter thus has to offer a higher wage in the core than in the periphery, making new technology less profitable to adopt in North than in South. The second major difference is that it avoids trade costs on its intermediate inputs should it choose the core, which is something a firm deviating to South has to live with.
We first analyse when production of the new goods is profitable in the periphery, but not so in the core. That is tantamount to asking when the adopting firm will be able to afford a wage that attracts workers from South’s A sector, whereas it cannot pay the wage needed to attract workers from North’s M sector. That is, when will \( \omega_{i+1}^* \geq 1 \) and \( \frac{\mu}{1-\mu} > \omega \), where \( \omega_{i+1}^* \) and \( \omega \) are given by (14) and (21), hold simultaneously? It is straightforward to show that \( \frac{\mu}{1-\mu} > \omega \) will hold if

\[
\beta_{i+1} > \theta \frac{\mu}{1-\mu} (1 - \alpha)^{\frac{1}{\tau}}.
\]

Parameter values satisfying this inequality ensure that the new technology will not be adopted in the core. They establish a lower bound on the productivity parameter, above which we can be sure the superior technology will be rejected by the leading country. It can still be adopted in South provided that

\[
\beta_{i+1} \leq \theta \frac{\mu}{1-\mu} \left( \frac{\mu}{1-\mu} \right)^{1-\alpha} (1 - \alpha)^{\frac{1}{\tau}} \left[ t^{\sigma-1-\sigma \alpha} (1 - \mu) + t^{1-\sigma-\sigma \alpha} \mu \right]^{\frac{1}{\tau}},
\]

which establishes an upper bound on \( \beta_{i+1} \). In Figures 3A and 3B below we plot the lower and upper bounds from (22) and (23) against \( t \) under two different sets of parameter values (reported in Appendix A.2).

**Figures 3A and 3B. Three adoption scenarios**
The horizontal line labeled $C$ is the core’s bound as given by (22). For values of $\beta_{i+1}$ above this line, adoption is not profitable in the core; if $\beta_{i+1}$ is below it, adoption in the core will be profitable. The periphery’s bound, from (23), is the curve labeled $P$. If $\beta_{i+1}$ is above this bound, the new technology will not be profitable to adopt in the periphery, whereas for values below the bound it will. We see that the derived bounds give rise to three qualitatively different adoption scenarios (referred to as "Technological Lock-In", "Leapfrogging", and "Leader adopts").

First, if $\beta_{i+1}$ is above the upper bound $P$ in Figure 3A, then the new technology will be rejected by both countries. It is simply not productive enough to offset the drawbacks of adoption. We refer to this scenario as technological lock-in and note that an increase in agglomeration benefits (higher $\alpha$; lower $\sigma$) will shift both bounds down, increasing the possibility of this outcome.\(^{17}\) The logic is straightforward. In the present framework the adopting firm loses intermediate demand from firms using the old technology in the core. Naturally, this disadvantage will be greater if intermediate demand is important ($\alpha$ is increased), and the new technology will have to be more productive to compensate for its loss. The effect of a change in $\sigma$ is similar. The lower this parameter is, the more important it will be for firms to have a large variety of intermediate goods available locally, and a potential adopter will be less inclined to deviate from the agglomeration. Again, the new technology will have to be more productive to be adopted. To conclude, the greater the agglomeration benefits that firms reap using the old technology, the less eager they are to give them up.

Second, if the value of $\beta_{i+1}$ is between the bounds in Figure 3A, adoption will be undertaken in South only and we may eventually get technological leapfrogging as illustrated in Figure 2B.\(^{18}\) Note that the existence of this possibility is independent of the level of $\theta$. However, a decrease in $\theta$ will shift both $C$ and $P$ down, increasing the likelihood of both regions rejecting the new technology. The less compatible the technologies are, the more productive the new technology has to be to be adopted. As we have already discussed, if $\theta$

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\(^{17}\)See Appendix A.3 for the analytical results.

\(^{18}\)As discussed in relation to figures 2A and 2B, the proviso is that the new technology is productive enough.
tends to zero, both bounds will also tend to zero and adoption will never be possible no matter how productive the new technology.

Finally, if $\beta_{i+1}$ is below the lower bound in Figure 3A, adoption will be profitable in both countries. In this case the new technology is adopted where the first adopting firm can afford to pay the highest wage. The reason is that the zero (pure) profit level of output is normalised to unity irrespective of where a firm adopts. The higher the maximum wage consistent with zero pure profits a firm is able to pay, the more attractive a location (see Fujita et al., 1999, p. 53 and p. 64). The vertical line $t \approx 2.50$ in Figure 3A shows where $\omega$ from (21) is equal to $\omega^*_{i+1}$ from (14); it partitions the area below $C$ into two parts. To the left of the line (in the area labeled "Leader adopts"), the adopting firm can afford to pay a higher wage should it choose the core rather than the periphery (that is, $\omega$ from (21) is greater than $\omega^*_{i+1}$ in (14)). Hence the new technology is adopted by the leading region, which preserves its front position in this case. To the right of the line (in the area labeled "Leapfrogging"), the wage is highest in the periphery and the first adopting firm locates there instead.19

In Figure 3B we see that a part of $P$ lies below $C$. In the area created by $C$ and that part of $P$ (labeled LA) adoption is only profitable in the core. While Figure 3B illustrates that a gap between the two bounds in (22) and (23) need not exist for intermediate values of $t$, we show in Appendix A.5 that it always does for small and large values of $t$. Thus, technological leapfrogging will always be possible if trade costs are either sufficiently low or sufficiently high.

5 Summary and Concluding Remarks

The advantages of locating production in an industrial centre are well established in the new economic geography. In this paper we point out a potential disadvantage: the risk of rejecting new technologies due to the agglomeration benefits associated with an existing technology. The Dutch rural region of Zaan, for example, had a very high concentration of the country’s woodworking industries (shipbuilding, mechanised sawing and timber trade) in the

19 We establish these differences in wages in Appendix A.4.
seventeenth and early eighteenth centuries (Vries and Woude, 1997). However, it lost its dominant position as a shipbuilding and lumber milling centre when new methods of ship construction, emphasising the use of iron, emerged. Furthermore, these newer methods of production would later allow the English to pioneer shipbuilding techniques that would lay the foundation for the innovations of the nineteenth century (Vries and Woude, 1997, p. 299). In this paper we introduce a model explaining how the strong links of the wood-based industries of Zaan prevented adoption of the new, and potentially superior, technology.

Our contribution to previous research on agglomeration economies and national/regional cycles in technological leadership is twofold. First, we show analytically that the risk of technological lock-in increases with the standard benefits of agglomeration highlighted in the new economic geography literature. The greater those benefits are, the smaller the incentive to give them up despite the productivity advantage a new technology may give adopting firms. Second, we model technological progress more delicately, allowing different technologies to be either completely or partially incompatible. This enables us to distinguish between major technological breakthroughs with little or nothing in common with old technology, and smaller improvements of existing technology. We show that the less compatible different technologies are, the greater the risk of rejecting new and more promising methods of production.

The Zaan region’s shipbuilding industry experienced slow technical development in the second half of the seventeenth century and thereafter. This puzzling technical conservatism may, according to Vries and Woude (1997), be attributed to the absence of guilds, which limited the transfer of technological knowledge. In this paper we point to an additional explanation. The industrial leadership of Zaan was built upon industries heavily dependent on wood. The strong inter-industry links among its wood-based industries may therefore have prevented the adoption of new shipbuilding techniques that were relatively more dependent on the use of iron. Such an explanation would seem even more plausible the less compatible the new methods of production were with the ones already in use.
Appendix

A.1 The original wage equation

The wage equation in the initial equilibrium (before the arrival of new technology) can be shown to be

\[ \omega^* = t^{\frac{\alpha}{1-\alpha}} \frac{\mu}{(1-\mu)} \times \left[ t^{\sigma-1} (1 - \mu) (1 - \alpha) + t^{1-\sigma} (\alpha + \mu (1 - \alpha)) \right] \frac{1}{\sigma(1-\alpha)}. \]  

(A.1)

To obtain the level of trade costs for which a relocating firm can offer a wage of unity, we set \( \omega^* = 1 \), \( \alpha = 0.6 \), \( \mu = 0.6 \), \( \sigma = 5 \) and solve (A.1) numerically for \( t \). There are two economically meaningful roots, \( t = 1.19 \) and \( t = 2.77 \). Our initial equilibrium will be stable if we choose \( t = 2.4 \).

A.2 Parameter values used for Figures 1-3

Figure 1: \( \alpha = 0.6, \beta_{i+1} = 0.95, \mu = 0.6, \sigma = 5, \theta = 1 \) (N1), and \( \theta = 0.6 \) (N2).

Figures 2A and 2B: \( \alpha = 0.6, \beta_{i+1} = 0.95 \) (Figure 2A), \( \beta_{i+1} = 0.9 \) (Figure 2B), \( \mu = 0.6, \sigma = 5, \theta = 1 \), and \( t = 2.4 \).

Figures 3A and 3B: \( \alpha = 0.6, \theta = 1, \mu = 0.6 \) and \( \sigma = 5 \) (Figure 3A), \( \mu = 0.55 \) and \( \sigma = 4 \) (Figure 3B).

A.3 Technological lock-in: analytical results

Using \( RHS \) to denote the right-hand side in (22), we have

\[ \frac{\partial RHS}{\partial \alpha} = \theta^{\frac{-\alpha}{1-\alpha}} (1 - \alpha)^{\frac{1}{2}} \left( \frac{\ln \theta}{\sigma - 1} - \frac{1}{\sigma (1 - \alpha)} \right) < 0, \]  

(A.2)

and

\[ \frac{\partial RHS}{\partial \sigma} = \theta^{\frac{-\alpha}{1-\alpha}} (1 - \alpha)^{\frac{1}{2}} \left( \frac{-\alpha \ln \theta}{(1 - \sigma)^2} - \frac{\ln (1 - \alpha)}{\sigma^2} \right) > 0. \]  

(A.3)

Proceeding in the same way with (23), we have

\[ \frac{\partial RHS}{\partial \alpha} = RHS \left( \frac{\ln \theta}{\sigma - 1} - \ln t - \ln \left( \frac{\mu}{1 - \mu} \right) - \frac{1}{\sigma (1 - \alpha)} \right) < 0, \]  

(A.4)
where \( RHS > 0 \) is the expression in the right-hand side of (23). The signs of these partial derivatives follow from the usual parameter restrictions: \( 0 < \alpha < 1, \) \( 0.5 < \mu < 1, \sigma > 1, t \geq 1, \) and \( 0 < \theta \leq 1. \) The final result is less clear-cut:

\[
\frac{\partial RHS}{\partial \sigma} = RHS \left( \frac{-\alpha \ln \theta}{(1 - \sigma)^2} - \frac{\ln (1 - \alpha)}{\sigma^2} + f(t) \right), \tag{A.5}
\]

where \( f(t) \equiv \frac{t^{\sigma-1}(1-\mu) - t^{1-\sigma} \mu}{[t^{\sigma-1}(1-\mu) + t^{1-\sigma} \mu] \sigma} \ln t - \frac{\ln[t^{\sigma-1}(1-\mu)]}{\sigma^2}, \) \( f(1) = 0, \) and \( f'(1) < 0. \) So \( \frac{\partial RHS}{\partial \sigma} > 0 \) when \( t \) is unity, but since \( f(t) \) is decreasing at that point we cannot be sure that \( \frac{\partial RHS}{\partial \sigma} \) remains positive as we move away from it. It can be shown that: i) \( t \geq \left( \sqrt{\frac{\mu}{1-\mu}} \right)^{\frac{1}{\sigma-1}} \) (\( \equiv t^* \)) is sufficient for \( f'(t) > 0 \) to hold, and ii) \( f(t^*) > 0. \) For all \( t \geq t^* \) we can thus be sure that \( \frac{\partial RHS}{\partial \sigma} > 0. \)

### A.4 Which country offers the highest wage?

Using (14) and (21) we have that the periphery’s wage is higher or equal to the core’s (\( \omega^*_{i+1} \geq \omega \)) provided that

\[
t^{\sigma-1-\alpha \sigma} (1-\mu) + t^{1-\sigma-\alpha \sigma} \mu - 1 \geq 0. \tag{A.6}
\]

Defining the left-hand side of (A.6) as a function of \( t, \) we want to analyse when \( f(t) \geq 0 \) holds, \( t \in [1, \infty). \) We have \( f(1) = 0 \) and \( f'(1) < 0 \) (the latter is true whenever \( 0.5 < \mu < 1). \) So the wages are equal when \( t \) is unity, but \( \omega^*_{i+1} < \omega \) for values of \( t \) in the vicinity of unity. Next we note that the middle term in (A.6) tends to zero as \( t \) tends to infinity (since \( \sigma > 1 \) and \( 0 < \alpha < 1) \), whereas the first one tends to infinity due to the no-black-hole condition (\( \sigma - 1 - \sigma \alpha > 0 \)). Therefore \( f(t) \to \infty \) when \( t \to \infty \) and it must be that \( \omega^*_{i+1} > \omega \) holds for high trade costs. Finally, we find a unique critical point, \( t^* = \left( \sqrt{\frac{(\sigma-1+\sigma \alpha)}{(\sigma-1-\sigma \alpha)(1-\mu)}} \right)^{\frac{1}{\sigma-1}}, \) which is a real root to \( f'(t) = 0 \) due to the no-black-hole condition (in fact, \( t^* > 1). \) Since the critical point is unique, and \( f(t) \) is continuous and has the properties mentioned above (i.e. \( f(1) = 0, f'(1) < 0 \) and \( f(t) \to \infty \) when \( t \to \infty), \) we know that there has to be an interior \( t \) for which the wages are equal. The dividing line where \( f(t) = 0 \) (\( \leftrightarrow \omega^*_{i+1} = \omega \)) is the vertical line in Figures 3A and 3B and we have solved for it numerically.
A.5 Is technological leapfrogging possible?

Leapfrogging is possible whenever the periphery’s bound \( P \) from (23) lies above the core’s bound from (22). We thus need

\[
\theta^{\alpha-\frac{1}{\sigma}} \left( \frac{\mu}{1 - \mu} \right)^{1 - \alpha} \left( t^{\sigma-1-\sigma\alpha} (1 - \mu) + t^{1-\sigma-\sigma\alpha} \mu \right)^{\frac{1}{\sigma}} > \theta^{\alpha-\frac{1}{\sigma}} (1 - \alpha)^{\frac{1}{\sigma}} \\
\left( \frac{\mu}{1 - \mu} \right)^{\sigma(1-\alpha)} \left[ t^{\sigma-1-\sigma\alpha} (1 - \mu) + t^{1-\sigma-\sigma\alpha} \mu \right] - 1 > 0.
\]

(A.7)

Defining the left-hand side of (A.7) as \( f(t), t \in [1, \infty) \), the analysis of (A.7) becomes somewhat similar to the analysis of (A.6). We note that \( f(1) > 0 \) holds as \( \mu > 0.5 \). It is also straightforward to show that \( f'(1) < 0 \). The gap between the two bounds hence exists for low values of \( t \), but it decreases as \( t \) increases. However, as the factor in front of the expression in square brackets is a positive constant, \( f(t) \to \infty \) as \( t \to \infty \), and we can be sure that the gap reemerges for high trade costs. That \( P \) does indeed lie above \( C \) for small and large values of \( t \) is thus established, and the possibility of technological leapfrogging exists.

References


