To Segment or Not to Segment Markets? A Note on the Profitability of Market Segmentation for an International Oligopoly

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Abstract

Recent research on endogenous market segmentation finds that a monopoly’s expected profit under perfectly segmented markets increases (relative to its profits under perfectly integrated markets) with exchange rate volatility. The firm thus has an incentive to make consumer resale increasingly difficult. We show that such an incentive may be absent for two firms competing in a Cournot fashion. While limitless consumer arbitrage forces a monopolist to deviate from its optimal pricing policies, it acts as a “disciplining device” helping the Cournot duopoly to approach and commit to the cartel solution in some markets. The firms’ total profit may hence be higher when they engage in integrated-market pricing and neither firm would have an incentive to take on additional costs to facilitate segmenting.

Keywords: arbitrage; Cournot duopoly; exchange rate volatility; market segmentation; third degree price discrimination

JEL Classification: D43, F31, L13

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I. Introduction and Previous Studies

Third degree price discrimination is when firms charge a different price to different consumer groups when selling the same good to the various groups. One frequently observed example is the price of movie tickets: students pay one price, retired people another, while the rest of us pay full price. Yet we all watch the same movie. It can also be that the ticket price is lower one day a week for all consumer groups (Mondays used to be a frequent choice in Swedish university town Lund), while the rest of the week everyone pays full price. Finally, third degree price discrimination can also be that different prices are charged in different geographical markets.¹

Why do firms segment markets? The natural answer is because it allows them to make larger profits. What does it take for such a pricing strategy to work? The answer is that the consumers can be grouped in terms of some easily determined characteristic, such as age or geographical location, and that arbitrage possibilities across the different consumer groups can be prevented. Hence, if you claim to be a student to get the lower ticket price for Academy Award®-winning The Hurt Locker, you normally have to show some kind of student ID. In the absence of such a control system everyone would claim to be a student or a black market would quickly emerge, where students buy tickets at the low price and sell them to consumers facing the higher price (just under-cutting that price and pocketing the difference). So when firms can segment markets perfectly arbitrage across different consumer groups is assumed to be prohibitively costly; when they engage in integrated-market pricing consumer arbitrage is assumed to have no cost and can only be prevented by charging the same price in each market. Interestingly, the existence of such arbitrage possibilities has in international trade theory mainly been an assumption, something lying outside the realm of firms’ decision-making.²

More recently, Friberg (2001) and Baldwin et al. (2007) make the actual degree of market segmentation an explicit decision of the firm. The basic idea is that as a monopoly makes

¹ Third degree price discrimination is the name given in industrial organisation (IO) to the practice described above. In international trade theory (ITT) it is more commonly known as perfect market segmentation. The opposite case is when firms charge the same price for all consumer groups, or set the same price in all markets. This is known as uniform pricing (IO), or perfect market integration (ITT). We use the terms interchangeably.
² See for example Markusen and Venables (1988) for an analysis of how trade policy affects welfare depending on whether markets are segmented or integrated. Smith and Venables (1988) investigate how the completion of
greater profits when it segments geographical markets, it has an economic incentive to invest
to make cross-border arbitrage more expensive, which gives the firm more leeway to segment
markets. Furthermore, sunk costs of market segmentation increases the expected profitability
of market segmentation (relative to market integration) the more volatile the exchange rate
between the markets is. If that is the case, then the creation of a monetary union would
contribute to less market segmentation by firms as exchange rate volatility disappears.

The aim of this note is to show that the incentive to make consumer resale more difficult may
vanish for a Cournot duopoly as the firms actually may earn higher profits when engaging in
integrated-market pricing. For a monopoly, profits are higher under perfectly segmented
markets relative to the profits earned under perfectly integrated markets. The reason is that
under perfect segmentation the firm sets marginal revenue equal to marginal cost in each
market, optimising total profits. If consumers are allowed to ship goods from the low-price
region to the high-price region, the firm is forced to deviate from this pricing schedule to
close the arbitrage possibilities (by raising the price in the low-price market and decreasing it
in the high-price market). Consumer arbitrage is thus a strait-jacket the monopolist would like
to rid itself of by investing to make resale between markets increasingly difficult.

With a Cournot duopoly, however, this is not the case. We know that in a standard single-
market homogenous good Cournot duopoly, the two firms would be better off if they could
form a cartel and cut back on production, increasing profits. The problem is that the cartel
solution is not stable as each firm has an incentive to increase production, but when both do
so they both lose. What consumer arbitrage does in a two-market setting is that it helps the
firms to commit to the cartel solution in one of the markets. Why? To close the arbitrage
possibilities the two firms cut back on production in the low-price market (which increases
the price there) and increase production in the high-price market (which lowers that market’s
price). This increases the profit in one market (where they cut back on production and
approach the cartel solution) and decreases it in the other. The exact circumstances under
which the net effect on total profits is positive will be discussed at length in the sequel. The
end result is that the ranking of profits for a Cournot duopoly may be reversed compared to
the monopoly. While a monopolist would have an incentive to make it more difficult for
consumers to resale goods, making it easier to segment markets and increase its profits, such

the internal market in the European Community would affect welfare by contrasting the polar cases of
segmentation and integration.
an incentive may be absent for a Cournot duopoly. Consumer arbitrage is a "disciplining device" that helps the two firms to behave more monopoly-like in one market. The industry would be worse off should resale be prevented.

The literature on the link between exchange rate variability and endogenous market segmentation is fairly new in international economics. Friberg (2001) establishes the insight that exchange rate volatility may increase the expected profits of market segmentation for a monopolist. Baldwin et al. (2007) develop a similar model and find some empirical support for its predictions when comparing bilateral exchange rate fluctuations and pricing within the deutschmark bloc relative to exchange rate fluctuations and pricing between Germany and its other European trading partners. The monopoly model is extended to a differentiated goods Bertrand duopoly in Friberg (2003), yielding similar results. Friberg and Martensen (2001) consider endogenous market segmentation for a Cournot duopoly, but their focus is on the importance of transportation costs (their model does not consider exchange rate variability). To the best of our knowledge, there is no prior study analysing the role of exchange rate volatility for the profitability of segmenting markets in a Cournot-setting.

The rest of this note is organised as follows. Section 2 outlines the basic structure of a partial equilibrium model and analyses the firms’ profits in the two polar cases of perfect segmentation and perfect integration. Section 3 explains why the Cournot oligopoly may be better off if they integrate markets. Some concluding remarks are given in section 4.

2 The Basic Framework

The world consists of two identical countries, Home and Foreign. The nominal exchange rate between Home and Foreign is denoted $e$ and it is defined in British terms, i.e. as Foreign currency units per unit of Home currency. A rise in $e$ thus corresponds to an appreciation of Home’s currency; a decline in $e$ a depreciation of Home’s currency. There is a Cournot duopoly located in Home, selling to both markets. The two firms use the same technology to produce a homogeneous good with constant marginal cost $c > 0$ (measured in Home’s currency). The supply of firm $i = 1, 2$ to Home is denoted $y_i$; $\pi_i$ denotes its profit earned from sales in Home. The corresponding variables when exporting to Foreign are denoted $y_i^*$ and $\pi_i^*$. The inverse demand functions in the two countries are:
(1) \[ P = a - (y_1 + y_2); \quad P^* = a - (y_1^* + y_2^*), \]

where \( a > 0 \), the slopes are normalised to unity and \( P^* \) is measured in Foreign’s currency. Firms incur a constant per unit cost \( t > 0 \) (measured in Home’s currency) when transporting goods between the countries. The optimisation problem of firm \( i \) is:

\[
\max_{\{y_i, y_i^*\}} P y_i - c y_i + \frac{P^* y_i^*}{e} - (c + t) y_i^*,
\]

i.e. firm \( i \) maximises total profits (measured in Home’s currency). As stated in the introduction we will consider two scenarios. We begin with the case of perfectly segmented markets and then proceed to perfectly integrated markets. In each case we look at the firms’ supply decisions and calculate their profits. We then analyse whether firms are better off under segmentation or integration.

2.1 Perfect market segmentation (no consumer arbitrage)

Without the possibility of consumer arbitrage (it is assumed to be prohibitively costly) and since marginal costs are constant, the firms can completely separate their supply decision for each market. In the Cournot-Nash equilibrium the supply of firm \( i \) to Home is \( y_i = \frac{a - c}{3} \), the price in Home is \( P = \frac{a + 2c}{3} \) and the profit earned by firm \( i \) is \( \pi_i = \frac{(a - c)^2}{9} \). For Foreign we have \( y_i^* = \frac{a - c(e + t)}{3} \), \( P^* = \frac{a + 2c(e + t)}{3} \), and \( \pi_i^* = \frac{(a - c(e + t))^2}{9e} \). Note that the firms’ de facto marginal cost of serving Foreign (measured in Foreign’s currency) is \( e(c + t) \). We assume that both markets always are served: \( a > c \) and \( e < \frac{a}{c + t} \equiv e^t_x \). Using the equilibrium prices it is straightforward to show that \( Pe = P^* \) if \( e = \frac{a}{a - 2t} \). To allow for this possibility, and recognising that exchange rates are strictly positive, we need \( a > 2t \) and this regularity.
condition is assumed to hold from now on. The level \( e = \frac{a}{a-2t} \) is also critical in determining in which direction arbitrage opportunities emerge once arbitrage is costless (as it will be in section 2.2): \( Pe < P^* \) if \( e < \frac{a}{a-2t} \) and \( Pe > P^* \) if \( e > \frac{a}{a-2t} \). That is, when Home’s currency is weak (\( e \) low), consumers in Foreign (whose currency then is strong) will rather buy the goods from Home than the goods sold in their own market. Similarly, if Home’s currency is strong (\( e \) high), then the consumers in Home will rather import the goods sold in Foreign. Firm \( i \)’s total profit under perfect segmentation is:

\[
\Pi_i^{PSM} = \frac{(a-c)^2}{9} + \frac{(a-e(c+t))^2}{9e}.
\]

The first term in the right-hand side is Home market profits; the second is profits on sales to Foreign. We next turn to perfectly integrated markets.

2.2 Perfectly integrated markets (limitless consumer arbitrage)

In this section we assume that there are no costs for “arbitrage entrepreneurs” to ship goods from one market to the other. If the firms still choose to segment markets, then arbitrage possibilities will emerge. As shown in section 2.1, which direction arbitrage occurs in depends on the exchange rate: if \( e \) is low (\( e < \frac{a}{a-2t} \)), then “arbitrage entrepreneurs” will buy the goods in Home and ship them to Foreign (since \( Pe < P^* \)); if it is high (\( e > \frac{a}{a-2t} \)) they will rather buy the goods in Foreign and ship them back to Home (as \( Pe > P^* \)). Should the firms not react to the threat of arbitrage, they run the risk of losing all their sales in one of the markets. Hence, in the former case (\( e \) low) the firms’ constraint will be \( Pe \geq P^* \) (the firms choose quantities to close the price gap); in the latter (\( e \) high) it will be \( P^* \geq Pe \). We will focus here on the low exchange rate case (the high exchange rate case is analogous and is reported in the Appendix A1). Firm \( i \) then faces the optimisation problem in (2) subject to \( Pe \geq P^* \), and its objective function now becomes:

\[3 \text{ If } a < 2t, \text{ then Home’s price can never be equal to Foreign’s when measured in a common currency (} P \text{ would be positive, } e \text{ negative and } P^* \text{ negative). This is clearly unsatisfactory from an empirical point of view.} \]
(4) \[ L = P y_i - cy_i + \frac{P^* y_i^*}{e} - (c + t)y_i^* + \lambda(Pe - P^*), \]

where \( P \) and \( P^* \) are given in (1) and \( \lambda \) is a Lagrange multiplier. The solution is

\[
y_i = \frac{e(3a - 2c - 2t) + a - 2c}{6(e + 1)}, \quad y_i^* = \frac{-2e^2(c + t) + e(a - 2c) + 3a}{6(e + 1)}, \quad P = \frac{2(a + c + e(c + t))}{3(e + 1)},
\]

\[
P^* = \frac{2e(a + c + e(c + t))}{3(e + 1)} \quad \text{and} \quad \lambda = \frac{a - e(a - 2t)}{2e(e + 1)}. \]

Note that \( P^* \) and \( Pe \) are related by the condition ensuring that there are no arbitrage possibilities, and that \( \lambda > 0 \) if \( e < \frac{a}{a - 2t} \equiv \lambda \), which is the same condition ensuring that \( Pe < P^* \). Obviously, should not the latter inequality be satisfied, then no arbitrage possibilities would exist in the first place from Home to Foreign and the constraint would not bind. Both markets are served if

\[
e < \frac{a - 2c + \sqrt{a^2 + 20ac + 24at + 4c^2}}{4(c + t)} \equiv e^*_x. \]

The total profit of firm \( i \) is:

\[
(5) \quad \Pi_{i}^{PSM} = \frac{2e^2(c + t)^2 - e(8ac - at - 4e^2 - 4ct) + 8a(a - c) + 2e^2 - 9at}{18(e + 1)}.
\]

As shown in the Appendix A1, the equilibrium values of quantities and prices do not change when the exchange rate is high and the constraint in the firms’ optimisation problem instead is \( P^* \geq Pe \). Hence, the solution for the firms’ total profit in (5) is obtained also in that case and we can use it for all permissible values of \( e \). We next turn to comparing the two scenarios.

### 2.3 Segmentation versus integration: when do the firms earn the largest profits?

The value of being able to segment markets for the firms is (taking the difference between (3) and (5)):

\[
(6) \quad \Pi_{i}^{Diff} \equiv \Pi_{i}^{PSM} - \Pi_{i}^{PIM} = \frac{e^2(a - 2t)(2a - t) + e(5at - 4a^2) + 2a^2}{18e(e + 1)}.
\]
Solving $\Pi^{\text{diff}} = 0$ for $e$ yields two real, positive roots: $e_1 = \frac{a}{a-2t}$ and $e_2 = \frac{2a}{2a-t}$. That the roots are positive numbers follows from the regularity conditions $a > 0$ and $t < \frac{a}{2}$ (since if $t < \frac{a}{2}$, then we must also have $t < 2a$). It is straightforward to show that $e_2 < e_1$ whenever $t > 0$, which is true as the firms’ transport costs between the markets are strictly positive. Interestingly, $e_1 = e_2$: The greatest of the roots is identical to the critical level that the exchange rate has to be lower than for the Lagrange multiplier $\lambda$ to be strictly positive, i.e. for the restriction to bind, in the case of a weak Home currency. Fig. 1 below is a plot of $\Pi^{\text{diff}}$ against $e$.

![Fig. 1. The difference in profits (duopoly in Home)](image1)

![Fig. 2. The difference in profits (monopoly in Home)](image2)

The possibility of being better off under perfectly integrated markets clearly exists: $\Pi^{\text{diff}} < 0$ for $e \in (e_2, e_1)$. For other values of $e$ the firms are better off under perfect segmentation. As a comparison, Fig. 2 illustrates the difference in profits had there instead been a monopoly located in Home serving both markets. For a monopolist the difference would be $\Pi^{\text{diff}} = \frac{(a-e(a-t))^2}{4e(e+1)}$, which is zero if $e = \frac{a}{a-t} \equiv e_m$ ($e_2 < e_m < e_1$) and strictly positive otherwise. Fig. 1 displays one note-worthy qualitative difference compared to Friberg (2001), Friberg (2003), and Baldwin et al. (2007), where the same difference is always non-negative.

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4 In the Appendix A1 we report the first-order conditions as they are instructive regarding how the adjustments in quantities occur to eliminate the price gap, which we will make use of in section 3.
5 Parameter values used to generate all the figures in the paper are reported in the Appendix A2.
(just as illustrated in Fig. 2)\footnote{Interestingly, the fact that a monopolist is better off under segmented markets is merely assumed to be true in Friberg (2001, p. 319): “To establish results we make the very weak assumptions that operating profits are higher under price discrimination than without…” , and he continues (ibid.): “By operating under segmented markets the firm gains an additional degree of freedom so that clearly $\Pi_s \geq \Pi_f$ for all levels of the exchange rate”. ($\Pi_s$ is the monopolist’s profit under segmented markets; $\Pi_f$ under integrated markets) A similar assertion is given in Friberg (2003, p. 657): “Clearly unconstrained profits are always higher than when prices are constrained to be equal across markets”. While these assertions may be true for a monopolist and a Betrand duopoly, it clearly is not the case here. We return to the reason for this in section 3.}: Each firm in a Cournot duopoly may actually earn higher profits when they are able to integrate markets. Their incentive to take on additional costs to make arbitrage more expensive is thus weaker for some levels of the exchange rate. This is more likely if the nominal exchange rate is a random variable following a probability distribution with much of its probability mass concentrated around the interval $(e_2, e_1)$ in Fig. 1. A second important observation from Fig 1., is that the thicker the tails of the probability distribution (i.e. the greater the exchange rate variability), the higher is the expected profit of being able to segment markets, just as in Friberg (2001). What is clear, however, is that for any given probability distribution, there is a counteracting force present in our setting, given by the expected loss of segmenting when $e \in (e_2, e_1)$, that makes segmenting less attractive to the two firms. Whether the magnitude of this expected loss is large enough to off-set the expected profits of segmenting for levels of $e$ outside the interval $(e_2, e_1)$, will ultimately depend on what is assumed about the exchange rate’s probability distribution. We next establish formally that the difference in profits can be strictly negative.

Rewriting (6) we have $\Pi_{\text{Diff}} = \frac{fe^2 + ge + h}{18e(e+1)}$, where $f \equiv (a-2t)(2a-t)$, $g \equiv a(5t-4a)$ and $h \equiv 2a^2$. From the regularity conditions $a > 0$ and $t < \frac{a}{2}$ it follows that $f > 0$, $g < 0$ and $h > 0$. Unfortunately, $\Pi_{\text{Diff}}$ is not strictly convex. To show that $\Pi_{\text{Diff}} < 0$ when $e \in (e_2, e_1)$, we need to proceed in a different way by using the following theorem:

Theorem A1. (Existence of extreme values on open intervals) If $f$ is continuous on the open interval $(a, b)$, and if $f(x) \to L$ as $x \to a^+$ and $f(x) \to R$ as $x \to b^-$ (where either or both of $L$ and $R$ may be finite numbers or $\pm \infty$), then:

I. If $f(x_0) > L$ and $f(x_0) > R$ for some $x_0 \in (a, b)$, then $f$ has a global maximum value on $(a, b)$.

6 Interestingly, the fact that a monopolist is better off under segmented markets is merely assumed to be true in Friberg (2001, p. 319): “To establish results we make the very weak assumptions that operating profits are higher under price discrimination than without…” , and he continues (ibid.): “By operating under segmented markets the firm gains an additional degree of freedom so that clearly $\Pi_s \geq \Pi_f$ for all levels of the exchange rate”. ($\Pi_s$ is the monopolist’s profit under segmented markets; $\Pi_f$ under integrated markets) A similar assertion is given in Friberg (2003, p. 657): “Clearly unconstrained profits are always higher than when prices are constrained to be equal across markets”. While these assertions may be true for a monopolist and a Betrand duopoly, it clearly is not the case here. We return to the reason for this in section 3.
II. If \( f(x_i) < L \) and \( f(x_i) < R \) for some \( x_i \in (a, b) \), then \( f \) has a global minimum value on \((a, b)\).

Proof: See the Appendix A3.

We now apply the theorem to the current setting. Our function is \( \Pi^{\text{Diff}}(e) = \frac{f e^2 + e g + h}{18e(e+1)} \), but its domain depends on which assumptions we make about the model’s parameters. The domain can be either \( 0 < e < e_x^l \) or \( 0 < e < e_x^r \), depending on whether \( e_x^l \) or \( e_x^r \) is smallest. Here, we analyse the case when \( 0 < e < e_x^l \) is the function’s domain.\(^7\)

First, it is straightforward to show that \( \Pi^{\text{Diff}} \to \infty \) when \( e \to 0^+ \) provided that \( h > 0 \), which we know is true: the \( y \)-axis is a vertical asymptote. Second, it is possible to show that \( \Pi^{\text{Diff}} \) is positive if \( e \) tends to \( e_x^l \) from the left.\(^8\) Using the notation in theorem A1, we have hence shown that \( L \) is positive infinity and \( R \) a positive number. If we now can find a candidate \( e^c \in (0, e_x^l) \) such that \( \Pi^{\text{Diff}}(e^c) < L \) and \( \Pi^{\text{Diff}}(e^c) < R \), we can be sure that a global minimum value exists. Clearly, either of the roots of the equation \( \Pi^{\text{Diff}} = 0 \), \( e_2 = \frac{2a}{2a-t} \) and \( e_1 = \frac{a}{a-2t} \), is such a candidate. Note that the minimum value need not be unique. However, it can be verified that \( \Pi^{\text{Diff}} \) has only one stationary point on \((0, e_x^l)\). To show this, we first solve \( \frac{d \Pi^{\text{Diff}}}{de} = 0 \), yielding two stationary points: \( e = \frac{a^2 + a\sqrt{(4a-t)(a-t)}}{3a^2 - 5at + t^2} \). The denominator is a strictly convex function of \( t \). Setting the denominator equal to zero and solving for \( t \) yield two real roots: \( t = \frac{d(5 \pm \sqrt{13})}{2} \). The smallest of these can be shown to be greater than \( \frac{a}{2} \), so the denominator is strictly positive under our regularity condition \( t < \frac{a}{2} \) (the same condition guarantees that the two stationary points are real numbers). Then the sign of the two stationary points depends on the sign of the numerator. It is easily shown that only

\(^7\) The table in the Appendix A4 gives the various critical parameter restrictions determining the relevant domain. In the text, we focus on case II in that table.
one, \( e = \frac{a^2 + a\sqrt{(4a-t)(a-t)}}{3a^2 - 5at + t^2} \), is strictly positive. The implication is that \( \Pi^{\text{Diff}} \) only has one stationary point on \((0, e^e)\), which we denote \( e^{SP} \). In addition, straightforward algebra shows that \( \frac{d \Pi^{\text{Diff}}}{de} \bigg|_{e^e} = -\frac{t(2a-t)^2}{12(4a-t)} < 0 \) (since \( t < \frac{a}{2} \), it must also be that \( t < 4a \)) and

\[
\frac{d \Pi^{\text{Diff}}}{de} \bigg|_{e^1} = \frac{t(a-2t)^2}{12(a-t)} > 0.
\]

That is, the slope of \( \Pi^{\text{Diff}} \) is negative at \( e^e \) and positive at \( e^1 \). As \( \Pi^{\text{Diff}} \) is continuous on its domain; that its slope changes sign from negative at \( e^e \) to positive at \( e^1 \); and given that is has a unique stationary point, then \( e^e < e^{SP} < e^1 \) must clearly be the case. Furthermore, considering that \( \Pi^{\text{Diff}} \) always crosses the horizontal axis twice (at \( e^e \) and \( e^1 \)); that it only has a single stationary point \( e^{SP} \) (with \( e^e < e^{SP} < e^1 \)), our global minimum value must be unique and negative, \( \Pi^{\text{Diff}} (e^{SP}) < 0 \).

To summarise, in Friberg (2001) and Baldwin et al. (2007) a monopolist can never be better off under perfectly integrated markets. Imposing a restriction on the monopolist’s optimisation problem, in the form of costless consumer arbitrage, forces the firm to deviate from its optimal pricing policy. The same conclusion is borne out in a Bertrand duopoly-setting (Friberg, 2003). Here, however, we have established that each firm’s profit can actually be higher under perfectly integrated markets. The implication is that the incentive to create barriers to arbitrage disappears for a Cournot duopoly for certain levels of the exchange rate. We next turn to the reason why the Cournot duopoly may be better off when they are able to engage in integrated-market pricing.

3. Consumer arbitrage as a disciplining device

Why is it that the two firms can benefit from being forced to engage in integrated-market pricing? It is well-known that each firm in a one-market, homogeneous good Cournot duopoly makes less profit than if the two firms could form (and commit to sustaining) a cartel. The problem is that the cartel solution is fundamentally unstable as each firm’s individual marginal revenue curve is above its marginal cost curve. Each firm thus has an incentive to

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8 Explicit parameter restrictions under which \( \Pi^{\text{Diff}} (e^e) > 0 \) has proved difficult to establish; see the Appendix A.5 for an alternative route.
increase its profit by increasing production, but when both do so they both lose (a prisoners’ dilemma).

Let us now return and analyse more closely what the firms actually do to close the arbitrage possibilities that emerge if equilibrium prices in the two markets differ and arbitrage is costless. In section 2.1 we noticed that $P_e < P^*$ when the exchange rate is low (Home’s currency is weak). We focus on this case here. If it is costless for consumers to ship goods, then arbitrage opportunities will occur as Foreign residents find it cheaper to buy the good directly from Home. We show in the Appendix A1 that in order to close the price gap the firms decrease their production in Home (raising $P$) and increase it in Foreign (lowering $P^*$). This adjustment in quantities proceeds until $P_e = P^*$ and the arbitrage opportunities vanish. Before this adjustment in quantities takes place each firm produces where its individual marginal revenue equals marginal cost in each market. However, at those levels of production the industry’s marginal revenue is below marginal cost and the sum of the two firms’ production levels is greater than the cartel’s: the industry’s profit is not maximised. The mutual decrease of production in Home thus helps the firms to approach the cartel solution there, increasing their profit from Home sales. In Foreign the firms will increase production to a scale where individual marginal revenue is below marginal cost, decreasing their profit there. We illustrate the adjustment in each market in Fig. 3 and Fig. 4 below.9

Fig. 3. Adjustments in Home

Fig. 4. Adjustments in Foreign

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9 The figures only show the general adjustment mechanism at work; while principally sound they are not numerically correct. Numerically correct examples can be derived using the parameter values presented in the Appendix A2, but the various effects were not illustrated as clearly.
The two figures share the following notation (a star denotes Foreign variables): $D$ is demand, given in (1); $MR$ is the industry’s marginal revenue ($a - 2Y$ in Home; $a - 2Y^*$ in Foreign), $mr$ is each firm’s individual marginal revenue ($a - 2y_i - y_j$ in Home and $a - 2y_i^* - y_j^*$ in Foreign; $i \neq j$), and $mc$ is the firms’ constant marginal cost. In Fig. 4, $t$ is the firms’ transportation cost and $e$ is the nominal exchange rate. The de facto marginal cost of serving Foreign is $e(c + t)$, measured in Foreign’s currency and taking the transportation cost into account. The superscript $B$ in each figure indicates equilibrium price and quantity before any adjustments take place; the superscript $A$ the new equilibrium price and quantity after the adjustments have eliminated the arbitrage possibilities. The cartel solution (not marked to avoid cluttering the figures too much) is where $MR = mc$ in each market.

In Home, see Fig. 3, we see that the firms initially (before the threat of arbitrage) set $mr = c$, selling $Y^B$ units. When arbitrage is allowed, the quantity is reduced to $Y^A$ (which raises $P$ from $P^B$ to $P^A$) and profits fall with area IV + V, representing the value of lost sales due to the quantity reduction. However, the output level $Y^A$ (that was formerly sold at price $P^B$) is now sold at price $P^A > P^B$, yielding a gain in profit represented by the area I + II + III. This gain dominates the loss and the total profit per firm on Home market sales increases. The firms thus approach the cartel solution, but what keeps them committed to the new equilibrium? Why do they not individually increase their output level? Inserting the equilibrium quantities after the adjustment in the expression for $mr$ above, it can be shown that $mr > c \leftrightarrow e < \frac{a}{a - 2t}$. We recognise the latter inequality as the restriction that ensures that $\lambda > 0$. That is, when the exchange rate is low, it is the very threat of arbitrage (enforcing the restriction to bind) that helps the firms to commit to the new equilibrium (producing a level of output closer to the cartel solution) in Home. In the absence of that threat, they would both increase production and make lower profits in that market. In Foreign, see Fig. 4, the firms produce where $mr^* = e(c + t)$ when they segment markets, selling $Y^{**B}$ units. When arbitrage is allowed, the opposite adjustment of quantities occurs in order to lower $P^*$ and not lose the sales there. We see that the firms are forced to produce an output level where $mr^* < mc^*$: it can be shown that $mr^* < e(c + t) \leftrightarrow e < \frac{a}{a - 2t}$. Again, the actual threat of arbitrage (a binding constraint) is what forces the firms to behave in this way in Foreign, but here their behaviour hurts them: They move further away from the cartel solution. The area...
IV represents the additional profit from increased sales; I + II + III is the lost profit of selling the old output level $Y^*_B$ at a lower price $P^*_A$. Clearly, the loss dominates the gain and the profit per firm decreases on Foreign sales. However, should the two firms not adjust their quantities in this way they would lose all sales in Foreign, which would be worse.

Considering the adjustments made on both markets the total net effect on the profit per firm is positive provided that $e \in (e_2, e_1)$, see Fig. 1 above. For values of $e$ outside this interval, the net effect is negative and the firms are worse off under perfectly integrated markets. To conclude: whereas consumer arbitrage is harmful to a monopoly in Friberg (2001) and Baldwin et al. (2007), it is a “disciplining device” for a Cournot duopoly that helps the firms to commit to the cartel solution in one of the markets. The implication is that the incentive to make resale between the markets more difficult may vanish for a Cournot oligopoly.

4 Concluding Remarks

Recent research in international trade theory has made a monopolist’s decision to segment markets endogenous, drawing on the assumption that a monopoly is better off under perfect segmentation. The monopolist thus has an economic incentive to make consumer resale more costly. Furthermore, exchange rate volatility increases the economic value of being able to segment markets relative to engage in integrated-market pricing. This note shows that the opposite may be true for an international oligopoly competing in a Cournot fashion. The reason is that as firms strive to eliminate the price difference that gives rise to arbitrage possibilities, they cut back on production in one of the markets and increase production in the other one. In doing so they approach the cartel solution in one market, increasing profits there, and go further away from it in the other market, decreasing profits there. The total net effect may be positive. It would thus seem that the threat of consumer arbitrage is a “disciplining device”, helping the firms to behave more monopoly-like in one market. The implication is that the incentive to make resale between the markets more difficult may vanish for a Cournot oligopoly. While the firms’ decisions whether to segment or integrate markets remain to be fully analysed, this note points out the possibility of being better off should they choose to integrate markets.
References


Appendix A1. The firms’ first-order conditions under perfect integration

Suppose first that the exchange rate is low when the firms segment perfectly (hence $P_e < P^*$). The first-order conditions associated with the Lagrangean function in (4) in the text are

$$\frac{\partial L}{\partial y_i} = a - 2y_i - y_i^{ex} - c - \lambda e = 0$$ and $$\frac{\partial L}{\partial \lambda} = a - 2y_i^{ex} - y_i^{ex} - c - t + \lambda = 0,$$

where $ex$ denotes each firm’s expectation about the other firm’s supply, $i \neq j$. Imposing symmetry in the first-order conditions ($y_i^{ex} = y_j$ and $y_j^{ex} = y_i$), and solving for the quantities give: $y_i = \frac{a-c-\lambda e}{3}$ and $y_i^* = \frac{a-e(c+t)+\lambda e}{3}$. The prices become $P = \frac{a+2c+2e\lambda}{3}$ and $P^* = \frac{a+2e(c+t)-2e\lambda}{3}$, where $\lambda = \frac{a-e(a-2t)}{2e(e+1)} > 0$ if $e < \frac{a}{a-2t} \equiv e_\lambda$. We first note that if $\lambda = 0$ (arbitrage is prohibitively costly and the constraint is not binding), then we retrieve the solutions in section 2.1. Given that $\lambda > 0$, these expressions reveal that the two firms raise the price in Home by cutting back on production there, whereas they lower the price in Foreign by increasing supply there. The difference in prices from section 2.1, which (if left unchecked) creates arbitrage possibilities when consumers face no resale costs, is thus eliminated by an appropriate adjustment of quantities.

If the exchange rate is high (and hence $P_e > P^*$ under perfect market segmentation), then the corresponding solutions are $y_i = \frac{a-c+\lambda e}{3}$, $y_i^* = \frac{a-e(a-2t)}{2e(e+1)}$, $P = \frac{a+2c-2e\lambda}{3}$, $P^* = \frac{a+2e(c+t)+2e\lambda}{3}$ and $\lambda = \frac{-a+e(a-2t)}{2e(e+1)} (\lambda > 0$ if $e > \frac{a}{a-2t})$. Hence the opposite adjustment of quantities is undertaken: the firms increase (decrease) production in Home (Foreign) until arbitrage possibilities are closed. Substituting the value of $\lambda$ into the expressions for the quantities and prices reveals that $y_i$, $y_i^*$, $P$ and $P^*$ are the same as in the low exchange rate case given in the main text. The expression for $\Pi^{PM}$ in (5) in the text is thus valid no matter in which direction arbitrage possibilities exist. Note, however, that $\Pi^{Diff} > 0$ whenever $e > \frac{a}{a-2t}$: this adjustment of quantities hurts them and the firms are better off under segmentation.
Appendix A2. Parameter values consistent with the figures

The following parameter values can be used for drawing figures similar to those in the text.

Figures 1 and 2: $a = 15$ and $t = 2$
Figure 3: $a = 15$; and $c = 2$
Figure 4: $a = 15$; $c = 2$; $t = 2$ and $e = 1.2$

Appendix A3. Proof of theorem A.1

We prove only part II, as a global minimum value is our concern here. We are given that there exists a number $x_1 \in (a, b)$ such that $f(x_1) < L$ and $f(x_1) < R$ (here $L$ and $R$ may be finite numbers or $\infty$). Given that $f(x) \to L$ as $x \to a^+$, there must exist a number $x_2 \in (a, x_1)$ such that $f(x) > f(x_1)$ for all $x \in (a, x_2)$. Similarly, there must exist a number $x_3 \in (x_1, b)$ such that $f(x) > f(x_1)$ for all $x \in (x_3, b)$. Hence $f(x) > f(x_1)$ at all points of $(a, b)$ that are not in the closed, finite subinterval $[x_2, x_3]$. As $f$ is a continuous function on $[x_2, x_3]$, it must by the Extreme Value Theorem have an absolute minimum value on that interval. Denote that point $m$. Since $x_1 \in [x_2, x_3]$, it must be that $f(m) \leq f(x_1)$ and hence that $f(m)$ is the minimum value on $(a, b)$.

Appendix A4. Various parameter restrictions

We need to be sure that both markets are always served so that the profit functions in equations (3) and (5) (and hence their difference) exist. In addition, there are other critical levels of the exchange rate and we need to relate them all to each other. We have $e_X^S \equiv \frac{a}{c + t}$ (the prohibitive level of $e$ under perfect segmentation) and

$$e_X^I \equiv \frac{a - 2c + \sqrt{a^2 + 20ac + 24at + 4c^2}}{4(c + t)}$$

(the prohibitive level of $e$ under perfect integration).

The levels of $e$ that solve $\Pi^{Diff} = 0$ are $e_1 = \frac{a}{a - 2t}$ and $e_2 = \frac{2a}{2a - t}$ ($e_2 < e_1$, see section 2.3 in the text). It is straightforward to show that the inequalities $e_X^I < e_X^S$ and $e_1 < e_X^I$ both hold.
if \( t < \frac{a-c}{3} \). Finally, \( e_2 < e_X^S \) if \( t < \frac{2(a-c)}{3} \). The right-hand side in the last inequality can be shown to be greater than \( \frac{a}{2} \) provided that \( a > 4c \), which we assume to be the case. (The assumption is rather weak as \( a \) is the vertical intercept of the inverse demand functions and \( c \) is the firms’ marginal cost in Home) Together with our regularity condition \( t < \frac{a}{2} \), the assumption \( a > 4c \) thus ensures that \( e_2 < e_X^S \) always holds. Also, we keep in mind that \( e_1 = e_X \), where \( e_X \) denotes the level of \( e \) that decides the direction of arbitrage if markets are segmented and arbitrage becomes costless. The following table (where \( I \equiv \frac{a-c}{3} \) and \( m \equiv \frac{a}{2} \)) summarises the relationships between the various levels of \( e \) above depending on the parameters \( a, c \) and \( t \):

<table>
<thead>
<tr>
<th>If the level of ( t ) is:</th>
<th>Then the levels of ( e ) are:</th>
<th>Arbitrage possibilities</th>
<th>Is ( \Pi^{\text{Diff}} &lt; 0 ) possible?</th>
</tr>
</thead>
<tbody>
<tr>
<td>Case I. ( l &lt; t &lt; m )</td>
<td>( e_2 &lt; e_X^S &lt; e_X^I &lt; e_1 )</td>
<td>From H to F (as ( Pe &lt; P^* ))</td>
<td>Yes, if ( e_2 &lt; e &lt; e_X^S )</td>
</tr>
</tbody>
</table>
| Case II. \( t < l < m \) | \( e_2 < e_1 < e_X^I < e_X^S \) | a) From H to F if \( e < e_1 \) (as \( Pe < P^* \))
                             |                                | b) From F to H if \( e_1 < e < e_X^I \) (as \( Pe > P^* \)) | Yes if \( e_2 < e < e_1 \) (that is, only in case a) |

In case I, the domain for \( \Pi^{\text{Diff}} \) is \( 0 < e < e_X^S \); in case II it is \( 0 < e < e_X^I \).

**Appendix A5. Establishing when \( \Pi^{\text{Diff}} (e_X^I) > 0 \)**

First, we know that \( \Pi^{\text{Diff}} \to \infty \) when \( e \to 0^+ \). We also know that \( \Pi^{\text{Diff}} = 0 \) has two real roots: \( e_1 \) and \( e_2 \), where \( 0 < e_2 < e_1 \). That is, \( \Pi^{\text{Diff}} \) does not cross the horizontal axis to the right of \( e_1 \): the graph of \( \Pi^{\text{Diff}} \) lies either above or below the \( x \)-axis when \( e > e_1 \). We also know that the slope at \( e_1 \) is positive, \( \frac{d \Pi^{\text{Diff}}}{de} \bigg|_{e_1} > 0 \): the graph of \( \Pi^{\text{Diff}} \) hence lies above the \( x \)-axis when \( e > e_1 \). Finally, \( e_1 < e_X^I \) if \( t < \frac{a-c}{3} \). Given that this inequality holds (this
corresponds to case II in the table in the Appendix A4), we can thus be sure that \( \Pi_{\text{diff}}^{(e'_X)} > 0 \).