Pay-What-You-Want in Competition

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Abstract

Pay-What-You-Want (PWYW) pricing schemes are popular in certain industries and not others. We model the seller’s choice of pricing scheme under various market structures assuming consumers share their surplus. We show that the profitability and popularity of PWYW depend not only on consumers’ preferences, but also on market structure, product characteristics and sellers’ strategies. While there is no equilibrium where PWYW dominates the market, given a sufficiently high level of surplus-sharing and product differentiation, it is chosen by the second mover to avoid Bertrand competition. The equilibrium results and their associated market characteristics are consistent with empirical examples of PWYW.


Keywords: Pay-what-you-want, competition, product differentiation, market behaviour, market structure.

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1 Introduction

Pay-What-You-Want (PWYW) is a pricing scheme in which a good is up for sale and the consumer, should he decide to buy, chooses the price to pay for it. Despite the expected free-riding behaviour by consumers, PWYW has in fact been adopted by numerous sellers. Arguably due to the extensive media coverage of Radiohead’s success, more and more instances of PWYW have come to light in recent years, particularly in the food, music and online retail industries (such as games and softwares).[1] Yet, despite the increasing use of PWYW in these industries, we still do not see its prevalence in many other sectors – fixed-price schemes still dominate.

The popularity of PWYW raises two questions. Firstly, how such a pricing scheme can exist: why do sellers adopt PWYW despite the possibility of getting no revenue, and accordingly, why do consumers pay a positive amount without having to do so? Although numerous studies have attributed the success of PWYW to consumers’ non-selfish behaviours, heterogeneity in preferences means that PWYW is prone to an adverse selection problem: selfish consumers self-select into the PWYW seller’s market and free-ride, causing the seller to make a loss. Second, the empirical examples of PWYW (listed in Appendix C) show a distinct cluster of sellers operating in imperfect competition against fixed-price competitors, selling non-resalable goods of low marginal cost, with some level of product differentiation. If indeed PWYW has the potential to generate more profits than fixed-pricing, why is PWYW still not commonly adopted in many other sectors?

This paper aims to address the above questions in an industrial organization

[1] In 2007, the band Radiohead released their album “In Rainbows” using PWYW. Hundreds of thousands of fans chose to pay a positive amount for the album, and the band in fact profited from this pricing format, making more money than from digital downloads of all their other studio albums combined (see http://musically.com/2008/10/15/exclusive-warner-chappell-reveals-radioheads-in-rainbows-pot-of-gold/ accessed 17-September-2014).
framework of competing pricing strategies. Previous studies of PWYW focus primarily on the role of consumer preferences to motivate above zero payments (see, for example, Kim et al. (2009) and Gneezy et al. (2012)), and theoretical models typically assume a monopolist seller (Chao et al., 2015; Fernandez and Nahata, 2009; Isaac et al., 2015; Mak et al., 2015). Our analysis instead focuses on the market structure and sellers’ strategies to explain the profitability and popularity of PWYW in certain industries and not others. Assuming a simple surplus-sharing mechanism to capture consumers’ social preferences, we first develop a benchmark model of a monopolist PWYW seller and analyse his profitability under this pricing scheme and as a fixed-price seller. Next, we analyse the outcome of a sequential competition between PWYW and fixed-price sellers of a homogeneous good to investigate the market characteristics for each equilibrium. Finally we introduce product differentiation to study its effect on the profitability of PWYW.

Chen et al. (2009) is a closely related paper which considers two competing sellers in a Hotelling model, where transport cost measures the degree of product differentiation. The authors find that in equilibrium, either both sellers choose PWYW, or both sellers choose fixed-pricing. While incorporating competition is an important next step in the evaluation of PWYW profitability, we make different assumptions regarding consumer decisions. Specifically, we assume that transport cost moderates product differentiation insofar as it determines the consumer’s choice of sellers, without creating heterogeneity in PWYW payment. We argue that this is a more realistic representation of a fair consumer who considers his surplus to be the pure consumption utility of a good less its marginal cost, without penalizing the seller for the extent of product differentiation. We show that these differences have important consequences for the resulting equilibrium outcomes.

Our model generates equilibrium predictions whereby either both sellers choose fixed-pricing or one of them chooses PWYW to avoid Bertrand competition. This is
the case both when products are homogeneous and differentiated, and is consistent with the instances of PWYW we see in the market which are in competition with other fixed-price sellers. In prior work, the success of PWYW has been attributed to preferences for fairness, reciprocity and social norms, or selfish and forward-looking consumers. In contrast, our model shows that even when consumers have social preferences, this is not enough to sustain a voluntary pricing scheme such as PWYW. The success of PWYW also requires certain market and product characteristics and the strategies of firms. However, when these conditions are fulfilled, PWYW is a simple and cheap strategy that a seller can adopt to escape the Bertrand trap. Next, we empirically analyse the existing examples of PWYW in the market and confirm the model’s predictions. The parameters that sustain the choice of PWYW by a seller include a low cost for the good, a high level (or not too high, in the case of product differentiation) of surplus-sharing, a low proportion of free-riders and an intermediate range of product differentiation – which are in line with the empirical examples of PWYW.

The rest of this paper will be structured as follows: Section 2 provides a brief review of the PWYW literature. The model is developed in Section 3, starting with the monopoly case followed by competition in section 4. Product differentiation is introduced in Section 5, while Section 6 presents a welfare analysis. Section 7 ties all the results together with the existing empirical examples of PWYW, and Section 8 concludes. All proofs are provided in the Appendix.

2 Literature Review

This paper adds to the growing literature on PWYW pricing schemes. Much of the PWYW literature on consumer preferences has proposed that consumers pay positive amounts because of selfish considerations to keep the seller in the market
(Mak et al., 2015) or other-regarding preferences. These include outcome-based social preferences such as altruism and inequity-aversion (Schmidt et al., 2014) or intention-based reciprocity (Regner and Barria, 2009; Regner, 2015). Alternatively, consumers pay a fair price to comply with social norms (Riener and Traxler, 2012; Jang and Chu, 2012; Chen et al., 2009). A self- or social-signalling mechanism such as that found in Gneezy et al. (2012) could additionally play a role in rationalizing the over-contribution in payment amount and the alternative opt-out behaviour.

Our analysis differs in that we focus on the market and seller characteristics that are likely to favour PWYW. We contribute to the literature by studying PWYW and fixed-pricing under different market structures including monopoly and price competition, while still incorporating social preferences in the consumer’s utility function through a surplus-sharing mechanism. Sellers’ entry into the market and their choice of pricing schemes are modelled in a sequential setting, with and without product differentiation. While our findings indicate that PWYW will never be the equilibrium mechanism chosen by all sellers, in certain cases it could be a profitable choice to avoid Bertrand competition against a fixed-price seller, or even as a monopolist. This outcome requires that a high enough level of surplus is shared with the seller, there is a low proportion of free-riders, the good has a low marginal cost, and, in the case of product differentiation, that the product is sufficiently differentiated.

Our results on consumer preferences and the good’s characteristics are in line with previous studies. A low marginal cost is a standard requirement for a seller to be able to sustain PWYW (Chao et al., 2015; Chen et al., 2009; Kim et al., 2013; Krämer et al., 2015), along with a sufficiently high adherence to fairness norms (see Armstrong Soule and Madrigal (2015); Chao et al. (2015), among others). The introduction of a charity component to PWYW in particular has mixed effects in the literature: while Gneezy et al. (2010) report higher profits relative to a charity
component on a fixed-price good, the argument outlined in \cite{Gneezy2012}, and modelled in \cite{Kahsay2015}, is that the pro-social component increases the perceived fair price of the good, thus potentially leading to an increase in the average price at the expense of purchase rate. On the other hand, the charity component may also invoke an increase in the image-sensitivity of the individual (see, for example, \cite{Gravert2014}). The success of Humble Bundle, the online game company which has consistently used PWYW, has been attributed precisely to its charity component.\footnote{See http://www.techdirt.com/blog/entrepreneurs/articles/20100716/17423610253.shtml, accessed 17-September-2014.}

In our model, the presence of a charity component is interpreted as an increase in the surplus-sharing norm in the target market. Consequently, this results in the higher profitability of PWYW (up to a point, if products are differentiated).

A sufficiently high level of product differentiation can be achieved by the seller through geographical differentiation by having a physical store. This allows more personal interaction with the buyer and hence lower social distance, which has previously been found to benefit PWYW sellers \cite{Kim2013}. Anonymity on the consumers’ side typically results in lower average prices, though it does increase purchase intentions \cite{Parvinen2013, Racherla2011, Regner2013}. However, \cite{Gneezy2012} find that anonymity increases average payments, appealing to the crowding-out and self-image explanations. When a transaction is monitored by the seller, an intrinsically motivated buyer may feel that his payment is made out of obligation, not fairness, crowding out its self-signalling value. Our model accommodates both possibilities using the surplus-sharing parameter, which can increase or decrease with anonymity depending on the presence or absence of intrinsic motivation in the buyer.

While most of the existing literature has focused on short-term experiments,
the few empirical studies that have followed PWYW sellers over the long run find that in general, though PWYW brings in more customers, the average prices paid decrease over time (Riener and Traxler 2012; Schons et al. 2014). The lack of research on PWYW’s feasibility in competition with other sellers is a clear gap in the literature that has so far only been addressed in Schmidt et al. (2014) and Chen et al. (2009). Using a laboratory experiment, Schmidt et al. (2014) find that in a competition between PWYW and fixed-price sellers, many consumers still prefer to choose the latter. Even when they do go to the PWYW seller, average prices are lower compared to a PWYW monopolist. On the theoretical front, Chen et al. (2009) study a model of competing sellers with horizontal product differentiation where transportation cost determines PWYW payment. While there are similarities in the monopoly results, we have different predictions regarding competition due to the different way in which the PWYW price is modelled, which is explained in more detail in Section 5. Both Schmidt et al. (2014) and Chen et al. (2009) predict an equilibrium in which both sellers choose PWYW, while our model predicts either a fixed-pricing equilibrium or one in which PWYW competes against fixed-pricing – empirically consistent with the majority of PWYW examples.

A summary of the relevant PWYW literature is provided in Table [I].
<table>
<thead>
<tr>
<th>Paper</th>
<th>Method</th>
<th>Charity</th>
<th>ERP</th>
<th>Min</th>
<th>Consumer Characteristics</th>
<th>Seller Characteristics</th>
</tr>
</thead>
<tbody>
<tr>
<td>Armstrong Soule and Madrigal (2015)</td>
<td>E</td>
<td>x</td>
<td></td>
<td></td>
<td>Anchors, social norms</td>
<td></td>
</tr>
<tr>
<td>Chao et al. (2015)</td>
<td>T</td>
<td>x</td>
<td></td>
<td></td>
<td>Fairness, social norms</td>
<td>Low cost</td>
</tr>
<tr>
<td>Chen et al. (2009)</td>
<td>T</td>
<td>x</td>
<td>x</td>
<td>x</td>
<td>Fairness, low willingness to pay</td>
<td>Low cost, competition, min price</td>
</tr>
<tr>
<td>Fernandez and Nahata (2009)</td>
<td>T</td>
<td>x</td>
<td></td>
<td></td>
<td>Positive valuation, social norms</td>
<td></td>
</tr>
<tr>
<td>Gautier and Klaauw (2012)</td>
<td>E</td>
<td>x</td>
<td>x</td>
<td></td>
<td>Selfish</td>
<td>Charity component</td>
</tr>
<tr>
<td>Gneezy et al. (2010)</td>
<td>E</td>
<td>x</td>
<td></td>
<td></td>
<td>Identity, self-image</td>
<td></td>
</tr>
<tr>
<td>Gneezy et al. (2012)</td>
<td>E</td>
<td>x</td>
<td></td>
<td></td>
<td>Identity, self-image</td>
<td></td>
</tr>
<tr>
<td>Gravert (2014)</td>
<td>E</td>
<td>x</td>
<td>x</td>
<td>x</td>
<td>Identity, self-image</td>
<td></td>
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<tr>
<td>Greiff et al. (2014)</td>
<td>T</td>
<td>x</td>
<td></td>
<td></td>
<td></td>
<td>Low asymmetry</td>
</tr>
<tr>
<td>Isaac et al. (2015)</td>
<td>T</td>
<td>x</td>
<td>x</td>
<td>x</td>
<td>Social norms</td>
<td>Minimum suggested price</td>
</tr>
<tr>
<td>Jang and Chu (2012)</td>
<td>E</td>
<td>x</td>
<td></td>
<td></td>
<td>Fairness, self-signalling, social norms</td>
<td></td>
</tr>
<tr>
<td>Johnson and Cui (2013)</td>
<td>E</td>
<td>x</td>
<td></td>
<td></td>
<td>Suggested price if close to internal reference price (IRP)</td>
<td></td>
</tr>
</tbody>
</table>

*aTheory or Empirical.  
bPWYW with charity component.  
cComparison with fixed-price schemes.  
dPWYW with external reference price.  
ePWYW with minimum price.
<table>
<thead>
<tr>
<th>Authors</th>
<th>Type</th>
<th>Factors</th>
<th>Notes</th>
</tr>
</thead>
<tbody>
<tr>
<td>Kahsay and Samahita</td>
<td>T</td>
<td>x</td>
<td>Self-image</td>
</tr>
<tr>
<td>(2015)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Kim et al. (2009)</td>
<td>E</td>
<td>x</td>
<td>IRP, fairness, satisfaction, price-</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>consciousness, income</td>
</tr>
<tr>
<td>Kim et al. (2013)</td>
<td>E</td>
<td>x</td>
<td>Reputation, low social distance,</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>low product value</td>
</tr>
<tr>
<td>Krämer et al. (2015)</td>
<td>E</td>
<td>x</td>
<td>High valuation, prosociality</td>
</tr>
<tr>
<td>León et al. (2012)</td>
<td>E</td>
<td>x</td>
<td>Low cost, promotional benefits</td>
</tr>
<tr>
<td>Mak et al. (2015)</td>
<td>T,E</td>
<td>x</td>
<td>Perceived dishonest</td>
</tr>
<tr>
<td>Parvinen et al. (2013)</td>
<td>E</td>
<td>x</td>
<td>Selfish, forward-looking</td>
</tr>
<tr>
<td>Racherla et al. (2011)</td>
<td>E</td>
<td>x</td>
<td>Threat of switching to FP</td>
</tr>
<tr>
<td>Regner and Barria (2009)</td>
<td>E</td>
<td>x</td>
<td>Anonymity</td>
</tr>
<tr>
<td>Regner (2015)</td>
<td>E</td>
<td>x</td>
<td>Anonymity</td>
</tr>
<tr>
<td>Regner and Riener (2013)</td>
<td>E</td>
<td>x</td>
<td>Anonymity</td>
</tr>
<tr>
<td>Riener and Traxler (2012)</td>
<td>E</td>
<td></td>
<td>Anonymity</td>
</tr>
<tr>
<td>Schmidt et al. (2014)</td>
<td>E</td>
<td>x</td>
<td>Anonymity</td>
</tr>
<tr>
<td>Schons et al. (2014)</td>
<td>E</td>
<td></td>
<td>One-shot transaction or price information</td>
</tr>
<tr>
<td>Schröder et al. (2015)</td>
<td>E</td>
<td>x</td>
<td>Warm glow</td>
</tr>
</tbody>
</table>

Table 1: List of PWYW studies.
3 Model

While the literature on PWYW consumers’ social preferences is extensive, a rich model of consumer behaviour capturing all the aspects previously mentioned, such as guilt, fairness and reciprocity, will unnecessarily complicate the model. This paper has a different goal and focuses instead on seller behaviour. From the point of view of the seller it is sufficient to observe and take as given that consumers are either free-riders or fair (who may pay more than the fair price or instead opt-out for any of the motivations above). These can be captured in a simple linear model of a consumer who maximizes his net surplus, as done in Chen et al. (2009); Cui et al. (2007); Economides (1986); Perloff and Salop (1985).

Each consumer is assumed to have unit demand. For simplicity, consumer i’s total utility from purchasing the good at price $p$ is assumed linear according to the following:

$$U_i = u_i - p.$$  

$u_i$ is the good’s consumption utility, or alternatively, i’s willingness to pay for the good. It is assumed to be uniformly distributed between zero and $k$ times the good’s constant marginal cost $c > 0$, which is public knowledge, so that $u_i \sim U(0, kc)$. $k$ is a scaling term which varies with the support of the consumption utility distribution. Moreover, $k > 1$ so that production of the good is efficient. The population size is normalized to 1, and the utility of no purchase is zero. We assume there is no fixed cost of production.

When the seller lets the consumer pay what he wants (PWYW), the behaviours of consumers vary. Assume a proportion $\theta$, $0 < \theta \leq 1$, are free-riders, who would

\footnote{In a later version obtained directly from Chen et al. (2013), a component for inequity-aversion is added to the utility function.}
always take the good for free. Previous studies have consistently found that a proportion of the population of individuals free-ride unconditionally, and that this behaviour type is stable (Kurzban and Houser 2005; Fischbacher et al. 2001). Hence, it is reasonable to assume that $\theta$ is an exogenous market parameter which can vary by country or industry. Cross-country variations in free-riding behaviour have been found in Kocher et al. (2008). It is also plausible to consider goods with charity component to attract fewer free-riders compared to other goods.

The remaining $1 - \theta$ consumers, however, are fair: they will pay at least $c$ and therefore will not purchase the good if their consumption utility $u_i$ is less than $c$. They will even split the surplus $u_i - c$ out of reciprocity for the seller having chosen a PWYW scheme (Schmidt et al. 2014), or any of the previously mentioned social preferences. Let $\lambda$ be the proportion of surplus shared with sellers, $0 < \lambda \leq 1$. This parameter represents the strength of social preferences in the economy, and can also be interpreted as an exogenous social norm – typically assumed to be 0.5 in an equal sharing rule, but in a richer and more generous economy the norm may be to give more and vice versa (see, for example, Gächter and Herrmann (2009) who find cross-cultural variations in reciprocity).

The fair consumer’s PWYW payment is therefore defined to be (Cui et al., 2007; Chen et al., 2009; Greiff et al., 2014)

$$p_i = c + \lambda(u_i - c).$$

4The analysis for $\theta = 0$ is straightforward and is left to the reader.
5Assuming consumers endogenously choose whether to free-ride or be fair towards the seller does not change our qualitative results. This analysis is provided as a robustness check in Appendix E.1.
6See also the literature on gift exchange, for example Fehr et al. (1998) where sellers offer high quality and consumers reciprocate by paying prices which are substantially higher than the sellers’ reservation prices.
7$\lambda = 0$ is simply the case of fixed-pricing at cost.
8Assuming $\lambda$ is heterogeneous has qualitatively similar results. The analysis is provided as a robustness check in Appendix E.2.
Substituting this payment into the utility function then gives the consumer’s PWYW utility:

\[ U_i = u_i - c - \lambda(u_i - c). \]

Observe that since \( \lambda \) and \( c \) are assumed exogenous, PWYW payment is deterministic and not obtained by utility maximization (as commonly modelled in the behavioural literature on consumer preferences, for example in Bénabou and Tirole (2006)). This means that given the seller offers PWYW, social norms dictate that consumers pay \( p_i \). If more than one pricing schemes are offered, consumers maximize utility by choosing the one which allows them to pay a lower price: be that the fixed price \( p \) or the PWYW price \( p_i \).

### 3.1 Monopoly

Under fixed-pricing (FP), a monopolist’s profit can be expressed as

\[ \pi_{FP} = \int_{p}^{kc} \frac{1}{kc} (p - c) du = (p - c) \left( 1 - \frac{p}{kc} \right) \]

using the familiar \((p - c)q\) notation. Performing the usual profit maximization calculation, we have optimal price, quantity and profit as follows:

\[ p_{FP} = \frac{c(k + 1)}{2}, \quad q_{FP} = \frac{k - 1}{2k}, \quad \pi_{FP} = \frac{c(k - 1)^2}{4k}. \]

Under PWYW, a monopolist’s profit can be expressed as

\[ \pi_{PWYW} = \theta \int_{0}^{kc} \frac{1}{kc} (-c) du + (1 - \theta) \int_{c}^{kc} \frac{1}{kc} (c + \lambda(u - c) - c) du \]

\[ = \frac{(1 - \theta)\lambda c(k - 1)^2}{2k} - \theta c. \]
Hence,

**Proposition 1.** *The monopolist will only choose PWYW when*

\[ \lambda > \hat{\lambda} = \frac{(k-1)^2 + 4\theta k}{2(1-\theta)(k-1)^2}, \]

*which increases with \( \theta \) and decreases with \( k \).*

PWYW will only be chosen if \( \lambda \), the level of surplus shared, is high enough, \( \theta \), the proportion of free-riders, is low enough, or \( k \), the scaling term corresponding to the support of \( u_i \), is high enough. This is illustrated in Figure 1. When the proportion of free-riders is high, PWYW profit is negative. As \( \lambda \) increases and \( \theta \) decreases such that

\[ \lambda > \frac{2\theta k}{(1-\theta)(1-k)^2}, \]

PWYW profit becomes positive, but still less than fixed-price profit. Only when \( \lambda \) exceeds the threshold \( \hat{\lambda} \) above will PWYW yield higher profit than fixed-pricing. As \( k \) increases, the \( \lambda \)-intercepts of these boundaries stay the same but the curves stretch to the right, increasing PWYW profit.

To illustrate why PWYW is rarely chosen by a monopolist, consider Fehr and Schmidt (1999, Table III) who estimate the proportion of individuals experiencing zero disutility from advantageous inequality to be around 0.3. Using this estimate for the number of free-riders \( \theta \) suggests that for the seller to choose PWYW over FP, even when \( \lambda \) is very close to 1, requires the good to be valued more than twice its cost on average \((k/2 > 2.40)\). As the average level of surplus-sharing decreases, the average valuation needs to increase. In a typical economy with a \( \lambda = 0.5 \) norm, PWYW profit will never exceed fixed-price profit.
4 Competition

Suppose now that there are two competing sellers selling the same product, and they can choose their preferred pricing schemes. Assume the product precludes resale. In stage 1, Seller A chooses either FP or PWYW. In stage 2, Seller B enters and chooses either FP or PWYW. In stage 3, any seller that chooses FP now chooses his price. If there are two FP sellers, the choice of price occurs simultaneously.

The sequentiality in entry closely models what we see in practice, whereby PWYW has commonly entered a market previously dominated by fixed-price sellers. Moreover, the simultaneity in price competition also captures the flexibility in prices which sellers can adjust dynamically. The full representation of the game and the resulting end nodes is shown in Figure 2. All decisions are common knowl-

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9With resale, a FP competitor or free-riding consumer can drive out the PWYW seller by buying a sufficiently large amount of the good at zero cost to resell them at a positive price.

10Letting sellers choose prices sequentially corresponds to a situation in which prices, once set, are fixed. An analysis is provided in Appendix A with similar results. Additionally, a second mover advantage may induce the incumbent to choose PWYW given an intermediate range of $\lambda$.  
At the end of Stage 3, the consumers make their purchase decisions. When both sellers choose PWYW, consumers randomize such that each seller gets half the market and shares the monopolist PWYW profit. When both sellers choose FP, consumers go to the seller with the lower price or randomize if prices are the same. Hence we assume that the usual Bertrand result applies where both sellers set \( p = c \) and make zero profit.

When there is one PWYW seller and one fixed-price seller, the free-riders will always take the good from the PWYW seller, while the fair consumers will go to the seller at which he will pay the lower price, be it the fixed price \( p \) or his PWYW price \( p_i \). Define
\[
    u_p = c + \frac{p - c}{\lambda}
\]
to be the consumption utility at which a fair consumer is indifferent between paying \( p_i \), his PWYW payment, and the fixed price \( p \). Therefore, when \( c \leq u_i < u_p \), he prefers to go to the PWYW seller, when \( u_i = u_p \) he is indifferent, and beyond \( u_p \) he is better off purchasing at the fixed price than sharing his consumer surplus with the PWYW seller. This is illustrated in Figure 3.

Clearly the fixed-price seller chooses the profit-maximizing price \( p \) taking into account...
Figure 3: Fair consumer’s action when PWYW and fixed-pricing both exist

account that this price will determine demand for both himself and his competitor. He will no longer get all the consumers with valuation greater than $p$ since the $\theta$ free-riders go to the PWYW seller. Out of the fair consumers, he will only get those with $u_i \geq u_p$ (see Figure 3). Hence the fixed-price seller will not set $p \geq c(\lambda k - \lambda + 1)$, as $u_p \geq kc$ and he would then get no customer. He will also not set $p \leq c$, as this will yield zero or negative profit. Therefore his fixed price will lie in $(c, c(\lambda k - \lambda + 1))$, and his profit can be expressed as

$$\pi_{FP} = \frac{1 - \theta}{kc} \int_{u_p}^{kc} (p - c) du.$$

The profit maximizing-price is thus

$$p^* = c \left( 1 + \frac{\lambda(k-1)}{2} \right)$$

and $u_p = c(k+1)/2$. Hence,

$$\pi_{FP} = (1 - \theta) \frac{\lambda c(k-1)^2}{4k}, \quad \pi_{PWYW} = (1 - \theta) \frac{\lambda c(k-1)^2}{8k} - \theta c.$$

The resulting profit for each seller is shown in Figure 2. To describe the equilibrium results, define the following:

**Definition 1.** In a separating equilibrium, one seller chooses PWYW and the other FP.

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11The set $(c, c(\lambda k - \lambda + 1))$ is non-empty since $\lambda > 0$ and $k > 1$. 

16
Definition 2. In a pooling equilibrium, both sellers choose the same pricing scheme, either PWYW or FP. Specifically, in the FP-pooling equilibrium, both sellers choose FP.

The equilibrium outcomes will now be summarized in Proposition 2, and illustrated in Figure 4.

Proposition 2. When two competing sellers choose pricing schemes sequentially and then enter into a simultaneous price competition, the subgame perfect equilibrium is either separating or FP-pooling. Specifically,

i when $\lambda > \lambda^*$, Seller A chooses FP, and Seller B chooses PWYW,

ii when $\lambda < \lambda^*$, Seller A chooses FP, and Seller B chooses FP,

iii when $\lambda = \lambda^*$, Seller A chooses FP, and Seller B randomizes between PWYW and FP,

where

$$\lambda^* = \frac{8\theta k}{(1-\theta)(k-1)^2}$$

which increases with $\theta$ and decreases with $k$.

We see that PWYW can be used as a strategy by the second mover to avoid Bertrand competition. Seller B choosing FP will lead to zero profit for both sellers. As long as $\lambda$ is sufficiently high or $\theta$ is sufficiently low, there is positive residual PWYW profit and Seller B will choose PWYW, with Seller A reaping the majority of the market profit. This is anticipated by Seller A, and therefore as a first mover he always chooses FP. Only when the PWYW profit becomes negative does Seller B prefer the Bertrand competition. All pure strategy equilibria are unique.

Note that $\lambda^*$ decreases as $k$, and hence the support of $u_i$, increases. As the good becomes more valuable to consumers, choosing PWYW becomes more profitable.
for Seller B as his residual profit (when Seller A has chosen FP) increases. Setting $\theta = 0.3$ (Fehr and Schmidt [1999]), the average valuation of the good needs to be at least 2.62 times its cost for PWYW to be chosen by the second mover, even when $\lambda$ is very close to 1 which is not often seen in practice. When $\lambda = 1/2$, the average valuation needs to be even higher (4.37) which may be less realistic. On the other hand, we see that for low values of $\theta$ it is possible to sustain a PWYW seller in competition for lower values of $\lambda$ compared to the monopoly situation.\footnote{This relationship is reversed if $\theta > (k - 1)^2/12k$ and $k < 13.93$. In this region it is more difficult for PWYW to survive competition, as the lower proportion of fair buyers contributes even lower profit due to the presence of the FP competitor. However, as can also be seen in Figure 4, the existence of this case also requires $\lambda \approx 1$ which is less common.} This is due to the opportunity cost of adopting FP: as a monopolist, choosing FP leads to positive profit, while the Bertrand competition profit is zero. Hence the switching point to FP occurs at a higher value of $\lambda$ as a monopolist than in competition.

In summary, no pooling equilibrium exists where both sellers choose PWYW. Instead, PWYW is used as a strategy by the second mover to avoid Bertrand com-

Figure 4: Subgame perfect Nash equilibria, $k = 5$
petition. Consequently, this makes PWYW a simple and cheap alternative to other costly marketing strategies such as differentiating products or introducing switching costs. For the first mover, the ‘threat’ of a competitor choosing PWYW is likewise beneficial in preventing the Bertrand equilibrium of zero profit.

5 Product Differentiation

Many PWYW examples can be found in markets with differentiated products, such as food, music and softwares (see the list of PWYW examples in Table 2 in the Appendix). While adopting PWYW seems to be more profitable for imperfect substitutes than homogeneous goods (we do not see a PWYW telecommunication company, for example), the adoption of PWYW does not quite reach the other extreme: products which are highly differentiated through exclusive brand names are still sold predominantly at fixed prices. In this section, we study a model of PWYW competition with horizontal product differentiation which can explain this finding.

Consumers are uniformly distributed along a Hotelling linear city of length 1. We continue to assume unit demand. For simplicity, and as commonly assumed in models of horizontal product differentiation including Hotelling (1929), consumption yields constant surplus $v = E(u) = kc/2$ as firms are assumed to be risk-neutral. This is a considerable simplification from the homogeneous product model with heterogeneous consumption utility studied in previous sections, however it facilitates the analysis to generate tractable results under product differentiation.

Consumers also pay a transportation cost $t > 0$, such that a consumer located at $x \in [0, 1]$ incurs disutility $tx$ if he purchases from Seller L located at 0, and $t(1 - x)$ from Seller R located at 1. Both sellers have the same profit and cost structures as before, with constant marginal cost $c$. We assume also that $v$, and hence $k$, is sufficiently large such that the market is fully covered: all consumers will purchase
a unit in equilibrium. Sellers choose their pricing scheme sequentially and prices are set at the end (simultaneously, if both sellers choose FP). With both sellers choosing FP, the equilibrium outcome is simple to calculate: both sellers set $p_L = p_R = c + t$ and get half the market with profits $\pi_L = \pi_R = t/2$. This result is intuitive: the higher the degree of differentiation, the higher the sellers are able to charge in mark-up over the cost of the good, while in the limit as $t \to 0$ we get the Bertrand equilibrium again.

Suppose now that both sellers adopt PWYW. When the consumer buys from a PWYW seller, his PWYW payment continues to be defined by the surplus-sharing mechanism as per Section 3: $p_i = c + \lambda(v - c)$. Note that we have assumed the surplus-sharing component is derived from the consumer’s total surplus from the good, not counting any reduction from transport cost. This is the case for a consumer who has to consume a good slightly different from his first choice, but upon arriving at the seller, in keeping with social norms pays according to the good’s pure consumption utility rather than discounting for how different it is from his actual taste.

For clarity in the analysis, assume no free-riders. The consumer’s utility from buying at Seller L is $U = v - tx - (c + \lambda(v - c))$, while from Seller R his utility is $U = v - t(1 - x) - (c + \lambda(v - c))$. As the payment for the good is identical at both

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13 It is straightforward to derive the required conditions: $k \geq 2 + 3t/c$ when $\lambda \in (0, 2/3]$, and $k \geq 2 + t/(c - c\lambda)$ otherwise.

14 The corresponding analysis for sequential price setting is provided in Appendix A.2. With positive transport cost, despite the second mover advantage it is still preferable for the first mover to choose FP, resulting in a FP-pooling equilibrium.

15 A consumer will be indifferent to purchasing at either seller if his utility from purchasing at Seller L, $U = v - p_L - tx$, equals the utility from purchasing at Seller R: $U = v - p_R - t(1 - x)$. His location is thus $x = (p_R - p_L + t)/(2t)$. Hence, from maximizing $\pi_L = (p_L - c)x$ with respect to $p_L$ and by symmetry, we get $p_L = p_R = c + t$ and $x = 1/2$.

16 The analysis with free-riders, which does not change the qualitative equilibrium results, is presented in Appendix E.3.
sellers, the indifferent consumer is located at $x = 1/2$ and each seller gets half the market with $\pi_L = \pi_R = \lambda(v-c)/2$. This is independent of the transport cost: when the consumer pays what he wants, his payment is deterministic. Consequently each seller always gets half the PWYW market profit regardless of the degree of product differentiation.

Suppose now that Seller L adopts PWYW and Seller R adopts FP. The indifferent consumer is now located at $x = (t + p_R - c - \lambda(v-c))/(2t)$. It is straightforward to derive the profit maximizing price of Seller R:

$$p_R = c + \frac{t + \lambda(v-c)}{2}$$

which implies

$$x = \frac{3}{4} - \frac{\lambda(v-c)}{4t}$$

and profits are

$$\pi_L = \frac{3\lambda(v-c)}{4} - \frac{\lambda^2(v-c)^2}{4t} \quad \pi_R = \frac{(t + \lambda(v-c))^2}{8t}.$$

For simplicity, assume that when the seller is indifferent between PWYW and FP he will choose FP. The equilibrium results are stated in the following proposition:

**Proposition 3.** When two competing sellers of differentiated products choose pricing schemes sequentially and then enter into a simultaneous price competition, the subgame perfect equilibrium is either separating or FP-pooling. Specifically,

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17 Abstracting from this assumption, in the special case where $\lambda = \frac{4c}{(k-2)c}$, both (FP,FP) and (FP,PWYW) are equilibrium outcomes. When $\lambda = \frac{2c}{(k-2)c}$, profits for all sellers at all end nodes equal $t/2$ and all of (FP,FP), (FP,PWYW), (PWYW,FP), (PWYW,PWYW) are equilibrium outcomes.
\( i \) when
\[
\frac{2t}{(k-2)c} < \lambda < \frac{4t}{(k-2)c},
\]
the first mover chooses FP and the second mover chooses PWYW.

\( ii \) otherwise, both sellers choose FP.

All pure strategy equilibria are unique. As the first mover always chooses fixed-pricing, when the surplus-sharing norm is low PWYW is attractive to consumers but yields low profit to the seller. On the other hand, with the assumption that the market is fully covered, the upper bound for \( \lambda \) is less than or equal to 1. Hence an extremely high surplus-sharing norm makes PWYW highly profitable per unit of the good, but demand is low since many customers would prefer purchasing at the (lower) fixed price. This is because the location of the indifferent consumer, \( x \), decreases with \( \lambda \). Therefore it is in the intermediate region of \( \lambda \) that a seller would choose PWYW against a FP competitor. Moreover, both upper and lower thresholds of this region are decreasing in \( c \) and it becomes more difficult for PWYW to be profitable for a higher cost item. For low values of \( \lambda \), as \( c \) increases the higher valuation for the good increases PWYW profit and results in the second mover choosing PWYW. However, when \( \lambda \) is high, the higher PWYW payment results in lower demand and fixed-pricing becomes more profitable.

The effect of varying the degree of product differentiation, as captured by the transportation cost \( t \), follows from the proposition above:

**Corollary 1.** Given \( \lambda \geq \frac{4t}{(k-2)c} \), when \( t \) increases to \( t' \in \left( \frac{\lambda(k-2)c}{4}, \frac{\lambda(k-2)c}{2} \right) \) the FP-pooling equilibrium becomes separating.

At low levels of product differentiation, demand for the PWYW seller is low. Consider the limiting case with homogeneous products: as \( t \to 0 \), the FP competitor can simply set \( p = c + \lambda(v - c) - \varepsilon \) and capture all consumers. Therefore, an
increase in $t$ serves to guarantee that some consumers will go to the PWYW seller as the location of the indifferent consumer $x$ moves closer to the FP seller. However, this increase in quantity becomes smaller as $t$ increases, with an upper bound at $3/4$, the limit of the location of the indifferent consumer when $t \to \infty$.

Note that the above increase in demand will only convince a FP second mover to switch to PWYW when the level of surplus-sharing norm is above the threshold given in Proposition 3 (where FP was chosen due to low demand). When the level of surplus-sharing is low such that PWYW results in high demand but is not sufficiently profitable, yet another increase in demand from product differentiation will not induce the FP seller to switch to PWYW as the amount paid by each consumer is still too low to overtake the profit increase as a FP seller.

It is worth discussing the key differences between this model and that in Chen et al. (2009). We have assumed here that the transport cost is not included in the surplus-sharing calculation: once the consumer ‘arrives’ at the PWYW seller, he considers his surplus to be the pure consumption utility less the cost of the good. In Chen et al. (2009), the consumer utility from purchasing at the PWYW seller is defined to be $U = v - tx - (c + \lambda(v - tx - c))$. When the consumer has the choice of PWYW and FP sellers, his surplus is defined to be $p_t - c$, where $p_t$ is the (fixed) price at which he is indifferent between buying from either seller. As a result, the location of the indifferent consumer and hence demand is independent of $\lambda$, the surplus-sharing parameter. The FP profit is lower compared to that derived here, giving rise to a PWYW-pooling equilibrium whenever $\lambda$ exceeds a threshold value which is increasing in transport cost, or a FP-pooling equilibrium otherwise. While the FP-pooling equilibrium is consistent with the results obtained here, as seen in the empirical examples it is rare to see a market dominated by PWYW. Moreover, the relationship between surplus-sharing, transport cost and the likelihood of PWYW in equilibrium is also not as straightforward as Chen et al. (2009) suggest:
while a higher level of surplus-sharing makes PWYW more profitable for the second mover, this is only true up to a point, beyond which higher surplus-sharing will drive away customers to the fixed-price competitor. Similarly, given a sufficiently high surplus-sharing norm, as the level of product differentiation increases, PWYW is more profitable for the second mover up to a point, beyond which FP would be preferred.

6 Welfare

In this section, we discuss the welfare implications of the various types of market structure taking into consideration the surplus of the consumers. We show that when PWYW arises in equilibrium, it may result in lower welfare for buyers.

Facing a monopolist seller, free-riders are always better off under PWYW than fixed-pricing, while for the fair consumers PWYW is preferred only if ‘not too much’ surplus is shared. With a norm of high surplus-sharing, fixed-pricing will be preferred. Overall, buyers will prefer PWYW if the level of surplus-sharing \( \lambda \) is less than some threshold value \( \tilde{\lambda} \). Since the monopolist seller only prefers PWYW if \( \lambda \) exceeds \( \hat{\lambda} \) as given in Proposition 1, it follows that:

**Proposition 4.** In an economy with a monopolist seller, PWYW will only be preferred by both the seller and buyers if

\[
\theta \leq \frac{(k-1)^2}{4}
\]

and \( \hat{\lambda} \leq \lambda \leq \tilde{\lambda} \), where

\[
\hat{\lambda} = \frac{(k-1)^2 + 4\theta k}{2(1-\theta)(k-1)^2} \quad \text{and} \quad \tilde{\lambda} = \frac{(k-1)^2(3-4\theta) + 4k^2\theta}{4(1-\theta)(k-1)^2}.
\]
A low proportion of free-riders $\theta$ is a necessary condition for PWYW to be preferred by both seller and buyers. Free-riders who have a low valuation for the good ($u_i < c$) and yet take it for free, incurring a cost $c$ to the seller, is a major contributor to dead-weight loss. With $\theta = 0.3$ \cite{Fehr,Schmidt}, PWYW being preferred by both seller and buyers requires that $k > 2.095$, or that the good is on average valued at 1.05 times its cost. This requirement ought to be fulfilled by most monopolist goods such as petrol or medicine, however the prevailing surplus-sharing parameter in the market may be too low for the seller. This results in FP being the preferred pricing scheme of the monopolist seller as explained in Section 3.

Under competition, while there is no PWYW-pooling equilibrium, one of the sellers may choose PWYW if the surplus-sharing norm $\lambda$ exceeds the threshold $\lambda^*$ (the north-west region in Figure 4). This avoids the Bertrand competition where both sellers set a price $p = c$ and get zero profit. Although the free-riders will prefer an equilibrium in which one seller offers PWYW, clearly the fair buyers prefer the FP-pooling equilibrium where they pay a fixed price of $c$, to the separating equilibrium where they either share their surplus or pay a higher fixed-price. Hence, the separating equilibrium will only be preferred if the proportion of free-riders is sufficiently high. However there is no compatible region in the $\lambda \theta$-plane in which the separating equilibrium is preferred by both sellers and buyers:

**Proposition 5.** In an economy with two competing sellers selling a homogeneous product, whenever the separating equilibrium obtains, it will never be preferred by buyers.

When products are differentiated as per the setting in Section 5, assuming all consumers are fair and have constant valuation of the good at $v = kc/2$, they will prefer the separating equilibrium if the size of the surplus shared is sufficiently low.
Specifically, when \( \lambda \leq \frac{2r}{(k-2)c} \), both sellers’ prices in the separating equilibrium are weakly lower than in the FP-pooling equilibrium. The indifferent consumer is now located to the right of \( x = 0.5 \). While some customers will pay more in transport cost to travel to the PWYW seller, the loss is made up by the savings made by those who still go to the closest seller and are now paying a lower price. However this is outside of the region in which PWYW is chosen by the seller as given in Proposition 3.

**Proposition 6.** In an economy with two competing sellers selling a differentiated product, whenever the separating equilibrium obtains, it will never be preferred by buyers.

### 7 Discussion and Empirical Observations

This paper studies the profitability of PWYW relative to fixed-pricing both as a monopolist and in competition, which has so far received little attention in the literature. In this section, the results from the analysis will be discussed in relation to the empirical examples of PWYW which are compiled in Table 2 in the Appendix.\(^{18}\) These examples come from previous academic literature (see Table 1) and following Google news alerts for “pay-what-you-want” from March 2014 to April 2015. While the list is not exhaustive and is skewed towards instances which generate a lot of publicity, it does offer some limited insight into the types of businesses that use PWYW. This also means that the proportion of sellers that are reported to have used PWYW for a limited time or have since discontinued PWYW at 32% is possibly understated, as a new seller opening a PWYW store would arguably generate more publicity. We therefore focus on the 77 current PWYW sellers in the discussion that

\(^{18}\)Refer to Appendix C for an explanation of how each example is classified according to its market and product characteristics.
follows.

Figure 5 show the distribution of current PWYW sellers across the various industries. The majority of PWYW businesses can be found in the retail sector (SIC Division G): in the food industry or selling digital products online. A significant number of sellers are in the service industry (Division I), including hotels and tourist attractions. The vast majority of sellers operate in a competitive environment. As shown in Figure 4, even for low levels of free-riding, a PWYW monopolist requires a higher level of surplus-sharing norm in the market relative to competition. Not surprisingly, empirical examples of PWYW monopolists are limited to the few football clubs or tourist attractions in our sample.

Figure 5: Market Structure of Current PWYW Sellers

As a way to increase the level of surplus-sharing in the market, many successful PWYW ventures have appealed to consumers’ generosity, for example by explicitly
stating that proceeds will be donated to charity (such as done by 14% of sellers). When the norm of surplus-sharing is high enough, in the competitive equilibrium a PWYW seller co-exists alongside a fixed-price seller. In particular, the first mover can avoid Bertrand competition by choosing a fixed price and ensuring that the second mover finds PWYW more profitable. This is seen in the trend of PWYW sellers’ entry into markets dominated by fixed-pricing, where they choose PWYW to avoid fierce competition and have instead appealed to the generosity of consumers. Correspondingly, a proportion of consumers do pay positive and high prices despite not having to do so (Gneezy et al., 2012; Kim et al., 2009, 2013; Riener and Traxler, 2012). For example, the company Activehours lets customers borrow funds against hours already worked with PWYW interest. It has recently entered a homogeneous, fixed-price market, and has instead chosen to let customers pay what they want in an effort to gain their trust and appeal to their generosity. Using PWYW is desirable both as a point of difference and to avoid the tough Bertrand-like competition in the market for lending. Furthermore, we do not see a market dominated by PWYW sellers competing against each other, consistent with the equilibrium predictions of Propositions 2 and 3. This signifies the strategic role played by firms’ choice regarding prices.

Our model also predicts the profitability of PWYW given a sufficiently high level of product differentiation. This is indeed what we see in Figure 6a, which confirms that the vast majority of PWYW sellers differentiate themselves either through geography or product characteristics as per Corollary 1. Figure 6b shows

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19 In the behavioural literature, for example, a charity component increases the perceived value of the good. One would then expect less free-riding and underpayment to occur as they result in a negative self-image in the consumer (Gneezy et al., 2012; Kahsay and Samahita, 2015). Consequently, the threshold level for surplus-sharing decreases. This is captured in our model more simply by assuming that the charity component attracts a higher level of surplus-sharing by consumers.

(a) Differentiation in either Geography or Product Characteristics

(b) Geographical Product Differentiation

(c) Differentiation in Product Characteristics

Number of current PWYW examples by SIC Division and product differentiation. Examples come from Table 2. SIC Divisions: “E” Transportation, Communications, Electric, Gas, And Sanitary Services, “G” Retail Trade, “H” Finance, Insurance, And Real Estate, “I” Services.

Figure 6: Product Differentiation of Current PWYW Sellers
many PWYW sellers in different industries operating from a physical store, thus allowing for geographical product differentiation. Moreover, the lower social distance generated by the personal nature of the transaction can serve to increase the surplus-sharing norm, relative to an anonymous online transaction [Hoffman et al., 1996; Regner and Riener, 2013]. As predicted in Corollary 1, when combined with high surplus-sharing, product differentiation makes PWYW increasingly profitable as the upper threshold value of surplus-sharing $\lambda$ increases. However, there also exist a significant number of PWYW sellers operating online. This can be explained by the low marginal cost of digital products. Proposition 3 predicts that as cost increases, the lower threshold value for generosity increases and it becomes more difficult for the separating PWYW and FP equilibrium to obtain. \(^{21}\) Not surprisingly, PWYW sellers of digital products, such as Humble Bundle and Storybundle, have operated successfully online. On the other hand, higher marginal cost items are less able to sustain the PWYW pricing model. This is seen in the examples of several hotels, such as Ibis, who have adopted PWYW for a period of time and then gone back to fixed-pricing.

When it comes to differentiation in product characteristics, it is clear that the majority of sellers in the retail and service industries do differentiate their products as shown in Figure 6c. The combination of product differentiation and high surplus-sharing is often achieved through various marketing strategies to promote the success of PWYW, for example by artist Amanda Palmer. She offers a differentiated product and directly appealed to fans to pay more, hence endogenously increas-

\[^{21}\text{The upper threshold also increases, meaning that an extremely high surplus-sharing norm, though unlikely, will drive fewer consumers away as the high cost justifies the high PWYW payment.}\]

\[^{22}\text{For homogeneous goods analysed in Section 4, a high value of marginal cost } c \text{ correspondingly makes a high value of } k \text{ unreasonable, due to consumers’ budget constraints. As the threshold value of surplus-sharing } \lambda, \text{ which must be exceeded for PWYW to be chosen, is decreasing in } k, \text{ a low marginal cost also indirectly makes PWYW more attainable.}\]
ing the level of surplus-sharing. Additionally, cafes or restaurants such as Seva Cafe attract generous consumers by advertising their charity connections. Given a sufficiently high surplus-sharing norm, PWYW is chosen by the second mover to avoid Bertrand competition with the FP incumbent (Proposition 3). An example of such entry behaviour is Kish restaurant (recounted in Kim et al. (2010)). As a new entrant in Frankfurt’s restaurant market, the owner decided to adopt PWYW on their lunch menu as it was found to be more profitable than fixed-pricing. This is not an isolated incident, as can be seen in the entry of many PWYW sellers into predominantly fixed-price markets in Table 2.

In most other markets where sellers face consumers with low generosity or when there is a high number of free-riders in the economy, it is not possible to sustain even one PWYW firm in equilibrium. This is what we see in instances such as the restaurant Five Loaves and Two Fish in China which discontinued PWYW after only a few months, having suffered big losses with 20% of customers eating for free. Consistent with our assumption of an exogenous surplus-sharing norm, the trend of successes and failures above has been attributed to cultural factors where PWYW does well in countries with high taxes and strong social welfare systems.

Using trust as a proxy, we find a 44% correlation between a country’s measure of trust and the presence of PWYW there. PWYW garners a lot of enthusiasm and publicity in the beginning, but in reality may be tough to sustain in the long term if customers have low levels of surplus-sharing. While data on PWYW duration

\[ \text{\textsuperscript{23}}\text{See the transcript of Amanda Palmer’s TED talk “The Art of Asking” (2013): } \text{http://www.ted.com/talks/amanda_palmer_the_art_of_asking/transcript}, \text{ accessed } 17-\text{September-2014.} \]

\[ \text{\textsuperscript{24}}\text{See } \text{http://www.bbc.com/capital/story/20140120-a-recipe-for-disaster}, \text{ accessed } 4-\text{March-2015.} \]

\[ \text{\textsuperscript{25}}\text{Ibid.} \]

\[ \text{\textsuperscript{26}}\text{Trust measure data comes from the World Values Survey Wave 6 (2010-2014) and the European Values Study Wave 4 (2008) question: “Generally speaking, would you say that most people can be trusted or that you need to be very careful in dealing with people?” A country’s trust level is calculated using the proportion of responders answering “Most people can be trusted.”} \]
is not freely available, we observe that many other businesses have only chosen
to experiment with PWYW through temporary promotions without committing to
permanent use. This is consistent with previous studies of PWYW which find that
average prices decline over time (Riener and Traxler 2012; Schons et al. 2014).

Finally, we also note the low possibility of resale of PWYW goods. As Table
2 shows, a large percentage of PWYW sellers sell experience goods with negligi-
ble marginal cost, such as theatre shows and tourist attractions, which have a low
resale possibility. Goods with higher marginal costs, such as food and drinks, are
often served directly to consumers which prevents a competitor from buying a large
volume and reselling it at profit. Goods that technically allow resale are limited to
digital products such as music and software, however in this case resale may not be
legal.

8 Conclusion

This paper aims to explain the mixed popularity of PWYW pricing schemes in
different sectors. Many PWYW examples can be found in monopolistically com-
petitive markets with some level of product differentiation, but few PWYW exam-
pies exist in perfect competition or as monopolists. While previous PWYW liter-
ature has studied consumers’ social preferences and their behaviour when facing a
PWYW seller, we focus on the seller’s choice between fixed-pricing and PWYW
pricing schemes while still retaining the social preference of consumers in a surplus-
sharing mechanism. Sellers’ strategies are studied in various types of markets where
entry occurs sequentially, to capture the commonly later entry of a PWYW seller
into a fixed-price dominated market. We show that the profitability of PWYW, and
hence its popularity, depends not only on the preferences of consumers but also
on the market structure, product characteristics and sellers’ strategies. There is no
equilibrium in which PWYW dominates the market. Given a sufficiently high level of surplus-sharing and product differentiation, PWYW can be chosen by the second mover as a simple strategy to avoid Bertrand competition. While the problem of adverse selection persists, in which PWYW attracts the free-riders and fair consumers with low valuation, in some cases this is still more profitable than entering into a price competition with the incumbent. If the level of surplus-sharing is too low, fixed-pricing dominates. These results are consistent with well-known empirical examples of PWYW. Welfare analysis shows that although PWYW is preferred by free-riders, fair consumers with high consumption utility will prefer paying a fixed price. As a result, the presence of a PWYW seller may reduce consumer surplus.

References


## Appendices

### A Sequential Competition

Suppose that competing sellers choose their prices sequentially. In stage 1, Seller A chooses either FP or PWYW. If FP is chosen, in stage 2 he sets his price. In stage 3, Seller B chooses either FP or PWYW, and if FP is chosen in stage 4 he sets a price. All decisions are common knowledge. This setting models situations whereby a PWYW seller enters a market dominated by a FP seller, whose price stays constant after the competitor’s entry. The full representation of the sequential game and the resulting end nodes is shown in Figure 7. At the end of the game, the consumers make their purchase decisions as previously described in Section 4 for homogeneous goods and Section 5 for differentiated goods.
A.1 Homogeneous goods

While there is no equilibrium in which both sellers choose PWYW, the pricing scheme is however used as an alternative to avoid Bertrand competition for either first or second mover, depending on the level of surplus-sharing. The full equilibrium outcomes are described in the following proposition:

Proposition 7. When two competing sellers choose both pricing schemes and prices sequentially, the subgame perfect equilibrium is either separating or FP-pooling. Specifically,

i when $\lambda > \hat{\lambda}$, Seller A chooses FP, and Seller B chooses PWYW,

ii when $\lambda \leq \hat{\lambda}$,

• when $\lambda > \lambda^*$, Seller A chooses PWYW, and Seller B chooses FP,

• when $\lambda < \lambda^*$, Seller A chooses FP, and Seller B chooses FP,
when $\lambda = \lambda^*$, Seller A randomizes between PWYW and FP, and Seller B chooses FP,

where $\hat{\lambda}$ and $\lambda^*$ are as previously given in Sections 3 and 4.

These regions are shown in Figure 8. When consumers share a sufficiently high proportion of surplus ($\hat{\lambda}$), Seller A can afford to set a fixed price and ensure that PWYW will be sufficiently profitable for Seller B. Otherwise, Seller B will always choose fixed-pricing, creating a Bertrand competition and capturing all profit. To avoid the Bertrand trap, if $\lambda$ is at least equal to $\lambda^*$ (or the number of free-riders is low), Seller A should choose PWYW: even though Seller B will still choose FP, there is still positive residual profit for the PWYW seller. All pure strategy equilibria are unique except the case where both sellers choose FP. In this case, Seller A will get zero profit regardless of what price is chosen, as it will be undercut by Seller B who will get a positive profit.
A.2 Product differentiation

As per Section 5, we continue to assume no free-riders and exogenous consumption utility $v = kc/2$. Without loss of generality, we assume that Seller R located at 1 is the first mover. Again, when a seller is indifferent between PWYW and FP he is assumed to choose FP. When products are differentiated and prices are chosen sequentially, in equilibrium no seller will choose PWYW:

**Proposition 8.** When two competing sellers of differentiated products choose both pricing schemes and prices sequentially, the subgame perfect equilibrium is FP-pooling.

Given the first mover’s set price, it is always optimal for the second mover to choose FP and undercut the first mover. In particular, in the (FP,FP) end node the first mover sets $p_R = c + 3t/2$, while the second mover sets $p_L = c + 5t/4$. On the other hand, when the first mover chooses PWYW, the second mover will find it more profitable to undercut the PWYW “price” and set a fixed price. As a result, the first mover is better off choosing FP and setting a sufficiently high price such that any residual demand still yields a higher profit.

B PWYW with Minimum Price

In this section we allow the PWYW seller to set a minimum price and analyse the equilibrium outcomes.

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27 Abstracting from this assumption, in the special case where $\lambda = \frac{5t}{5k - 2}$, both (FP,FP) and (FP,PWYW) are equilibrium outcomes. Additionally, if $\lambda = \frac{3t}{5k - 2}$, both (PWYW,FP) and (FP,FP) are equilibrium outcomes.
B.1 Monopoly

Suppose now that the PWYW seller decides to impose a minimum price $p_m$ to reduce the problem of free-riders. Consequently, the free-riders will now only buy at $p_m$ if their consumption utility exceeds this minimum price.

For the fair consumers, we assume that the introduction of the minimum price does not alter the surplus-sharing mechanism in the utility function. Define

$$u_j = c + \frac{p_j - c}{\lambda}$$


to be the consumption utility at which a fair consumer is indifferent between paying $p_i$, his PWYW payment, and some fixed price $p_j$ (that is, $p_i = p_j$). Therefore, whenever $u_i < u_m$, $p_i < p_m$ and the minimum price will be binding. The fair consumers will continue to buy and pay $p_m$, and only when their consumption utility is lower than $p_m$ will they stop buying. As long as the minimum price is not binding, they will continue to pay $p_i$. Note that the minimum price will always bind if $p_m \geq c(\lambda k - \lambda + 1)$. This information is summarized in Figure 9.

\begin{figure}[h]
\centering
\begin{tikzpicture}[scale=0.8]
\draw[->] (0,0) -- (5,0);
\draw[->] (0,0) -- (0,1);
\draw[->] (5,0) -- (5,1);
\node at (0.5,0) {$0$};
\node at (2.5,0) {$c$};
\node at (4.5,0) {$p_m$};
\node at (5.5,0) {$u_i$};
\node at (0,0.5) {$no$ \hspace{0.5cm} purchase$};
\node at (2.5,0.5) {$pay$ \hspace{0.5cm} $p_m$ \hspace{0.5cm} $pay$ \hspace{0.5cm} $p_i$};
\node at (4.5,0.5) {$u_m$ \hspace{1cm} $kc$};
\end{tikzpicture}
\caption{Fair consumer’s action when a PWYW seller uses a minimum price}
\end{figure}

The PWYW seller maximizes his profit by choosing the optimal minimum price. Note first that when $p_m = 0$, this is simply the original PWYW case. As $p_m$ increases, the free-riders start paying a positive amount, but as long as $0 < p_m < c$ the seller is still losing money to the free-riders while profit from the fair consumers is unaffected. At $p_m = c$, the seller breaks even with the free-riders while still cap-
turing all profit from the fair consumers, resulting in

\[ \pi_m(c) = \frac{(1 - \theta)\lambda c(k - 1)^2}{2k}. \]

Hence, we make the following observation:

**Observation 1.** The original PWYW scheme is always dominated by PWYW with a minimum price of \( p_m \leq c \).

As a monopolist, it is always more profitable to set a minimum price, even when it is less than \( c \), than letting consumers pay what they want freely. The fair consumers will not be affected, while the seller will stop incurring loss from the free-riders. Indeed, with an optimal minimum price, PWYW can be even more profitable than fixed-pricing.

**Proposition 9.** The optimal monopolist minimum price is

\[
p_m = \begin{cases} 
\frac{c(k + 1)}{2}, & \text{if } \lambda \leq \frac{1}{2} \\
\frac{c(1 + \frac{\theta \lambda(k - 1)}{1 + \frac{\theta}{2} - 1})}{2}, & \text{if } \lambda > \frac{1}{2}
\end{cases}
\]

with \( c < p_m < kc \) when \( \lambda \leq 0.5 \), and \( c \leq p_m < \frac{c(k + 1)}{2} \) when \( \lambda > 0.5 \). This yields a profit which is weakly greater than the fixed-price scheme and strictly greater than pure PWYW without a minimum price.

When \( \lambda \leq 1/2 \), the level of surplus shared is low and setting the minimum price equal to the profit-maximizing fixed price is optimal. In particular, assuming an equal sharing rule, the optimal pricing scheme is simply the fixed-price scheme. Profit is also higher than the original PWYW scheme (since \( \lambda \leq 1/2 \) and \( \theta > 0 \)). However, when \( \lambda > 1/2 \), it pays to lower the minimum price to let those consumers with higher valuations share their surplus with the PWYW seller. There is no loss
from free-riders and the seller still captures some of the consumer surplus. As $\lambda$ increases, the optimal minimum price continues to decrease to let the increasingly generous consumers make their payments, down to $c$. However, as the proportion of free-riders $\theta$ and demand parameter $k$ increase, the optimal minimum price also (naturally) increases to offset the loss from free-riders and take advantage of the higher willingness to pay.

### B.2 Competition

We next analyse what happens in competition when the PWYW seller can institute a minimum price. The game tree is shown in Figure 10. Each seller enters sequentially and chooses his pricing scheme, and in the last stage both sellers choose their minimum and/or fixed prices simultaneously.

At the (FP,FP) end node, buyers choose the seller with the lower price, or randomize if indifferent. At the (PWYW,PWYW) end node, consumer behaviour is described as follows: a consumer will only buy the good if his consumption utility exceeds the minimum price. Since free-riders simply pay the minimum price, they will always go to the seller with the lower minimum price or randomize if indifferent. For the fair buyer, as long as the minimum price is not binding, he will continue
to pay $p_i$. However, when the minimum price at seller $j$, $j = \{A, B\}$, is binding, he will only buy at the minimum price $p_j$ if $u_i \geq p_j$. Similar to the monopoly case, the minimum price will always be binding if $p_j \geq c(\lambda k - \lambda + 1)$. Given this payment schedule, he will go to the seller which lets him pay the lower price or randomize if indifferent.

For example, Figure 11 shows the case where $p_A < p_B < c(\lambda k - \lambda + 1)$ (hence the presence of $u_A$ and $u_B$ in the diagram). A fair buyer with $p_A < u_i < u_A$ will pay the minimum price $p_A$ at Seller A. When $u_A < u_i < kC$, $p_A$ is not binding and hence going to Seller A entails paying $p_i = c + \lambda(u_i - c)$. However, Seller B has a higher minimum price. Hence at $u_A < u_i < p_B$ the buyer cannot and will not go to Seller B, and will continue to choose Seller A. When $p_B < u_i < u_B$, the buyer can go to Seller B and pay $p_B$, but it is better for him to go to Seller A and pay $p_i$ since $p_i < p_B$ (since $u_i < u_B$). Therefore, when $u_A < u_i < u_B$ the fair buyers will still go to Seller A. When $u_B < u_i < kC$, going to either seller means paying $p_i$, so the fair buyer will randomize.

![Figure 11](image)

**Figure 11:** Fair consumer's action when two PWYW sellers use minimum prices.

At end nodes where one seller chooses PWYW and the other FP, buyers continue to choose the seller at which he pays the lower price. At the PWYW seller, this price is either $p_i$ or the minimum price, if it is binding, while at the FP seller he pays the fixed price.

For example, Figure 12 shows the case where Seller A chooses PWYW and Seller B has a fixed price, with $p_A < p_B < c(1 + \lambda)$ (hence the presence of $u_A$ and $u_B$ in the diagram). A fair buyer with $p_A < u_i < u_A$ will pay the minimum price $p_A$ at Seller A. When $u_A < u_i < kC$, $p_A$ is not binding and hence going to Seller A entails
paying $p_i = c + \lambda(u_i - c)$. Seller B has a (higher) fixed price $p_B$, so the fair buyer will only go to Seller B if $u_i \geq p_B$. Hence at $u_A < u_i < p_B$ the buyer cannot and will not go to Seller B, and will continue to choose Seller A. When $p_B < u_i < u_B$, the buyer can go to Seller B and pay $p_B$, but it is better for him to go to Seller A and pay $p_i$ since $p_i < p_B$ (since $u_i < u_B$). Therefore, when $u_A < u_i < u_B$ the fair buyers will still go to Seller A. When $u_B < u_i < kc$, $p_i > p_B$ and hence the buyer will choose the fixed-price Seller B.

![Figure 12: Fair consumer’s action when PWYW with a minimum price and fixed-pricing both exist](image)

The resulting equilibrium can be summarized in the following proposition:

**Proposition 10.** Consider two sellers of a homogeneous good competing using either fixed-pricing or PWYW with a minimum price, where pricing schemes are chosen sequentially and prices are chosen simultaneously. In equilibrium either both sellers choose FP or both sellers choose PWYW with a minimum price of $c$.

No pure strategy Nash equilibrium exists in the subgames where PWYW and FP both co-exist in the market. Either both sellers choose FP, or both sellers choose PWYW with a minimum price of $c$. Clearly the PWYW equilibrium is preferred by both sellers since it gives positive profit compared to the zero profit Bertrand equilibrium.

### C PWYW Examples

In Table 2 a summary of anecdotal evidence of PWYW is provided, based on media coverage (current as at 15 April 2015). These are the most popular sellers found
by following news alerts for “pay-what-you-want” since March 2014 and using examples commonly quoted in previous academic literature.\(^{28}\)

Each business has been categorised according to the *Standard Industry Classification (SIC) Division*, which broadly describes its industry, and *Major Group*, which further categorises the seller according to the type of product sold.\(^{29}\) Under *Market Structure*, a seller is classified as operating in Competition, except for football clubs, museums and other tourist attractions. These have been classified as Monopolists, where we have defined the market level to be the seller’s city of operation. A business has *Geographical Product Differentiation* if it has a physical location, in contrast to online sellers. *Differentiation in Product Characteristic* refers to whether the product sold has a close substitute. While this is a coarse way to capture product differentiation, no established measure currently exists. Products that are classified as undifferentiated and have close substitutes include ridesharing, loan interest, money transfer service and a tax software. A product is classified as having no *Marginal Cost* if it is sold online or falls under one of the following categories: theatres, movie shows, art galleries, tourist attractions, and football games. A product is *Resalable* if it is not an experience good, which also excludes food and drinks, ridesharing, hotel stays, and tourist attractions. This leaves all online commodities such as softwares, music and games in the (perhaps not legally) resalable category.

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\(^{28}\)The owner of One World Cafe, one of the most popularly cited examples of PWYW, has now turned to consulting other business owners and encouraging the use of PWYW in a large number of other restaurants. These are excluded from the table, since they focus specifically on religious or community aspects. The full list can be found on [http://www.oneworldeverybodyeats.org/other-community-cafes/](http://www.oneworldeverybodyeats.org/other-community-cafes/), accessed 17-September-2014.

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*a* Standard Industry Classification  
*b* Geographical product differentiation.  
*c* Differentiation in product characteristics.  
*d* Marginal cost.  
*e* Resale possibility.  
*f* Explicit charity component.
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Table 2: PWYW examples.
D Proofs

D.1 Proposition 1

\[ \pi_{\text{PWYW}} = \frac{(1 - \theta)\lambda c(k-1)^2}{2k} - \theta c > \frac{c(k-1)^2}{4k} = \pi_{FP} \]

implies

\[ \lambda > \frac{(k-1)^2 + 4\theta k}{2(1-\theta)(k-1)^2} = \hat{\lambda}. \]

It is straightforward to derive the following:

\[ \frac{d\hat{\lambda}}{d\theta} = \frac{(k+1)^2}{2(1-\theta)^2(k-1)^2} > 0 \quad \frac{d\hat{\lambda}}{dk} = -\frac{2\theta(k+1)}{(1-\theta)(k-1)^3} < 0. \]

D.2 Proposition 2

From Figure 2, when Seller A has chosen FP, at B.2 Seller B will only choose PWYW if

\[ \frac{(1 - \theta)\lambda c(k-1)^2}{8k} - \theta c > 0, \text{ or } \lambda > \frac{8k}{(1-\theta)(k-1)^2} = \lambda^*. \]

If Seller A chose PWYW, Seller B will always choose FP at B.1. Hence, given \( \lambda \), it is straightforward to derive the equilibrium actions for both sellers, which are summarized in Proposition 2.

It is also straightforward to derive the following:

\[ \frac{d\lambda^*}{d\theta} = \frac{8k}{(1-\theta)^2(k-1)^2} > 0 \quad \frac{d\lambda^*}{dk} = -\frac{8\theta(k^2 + k - 1)}{(1-\theta)(k-1)^4} < 0. \]

D.3 Proposition 3

Suppose the first mover A has chosen PWYW. The second mover B will always choose FP since

\[ \pi_B(PWYW, PWYW) = \frac{\lambda(v-c)}{2} \leq \frac{(t + \lambda(v-c))^2}{8t} = \pi_B(PWYW, FP). \]
On the other hand, if the first mover has chosen FP, the second mover will choose PWYW if

$$\pi_B(FP, PWYW) = \frac{3\lambda(v-c)}{4} - \frac{\lambda^2(v-c)^2}{4t} > \frac{t}{2} = \pi_B(FP, FP)$$

which will be the case if $$\lambda \in \left(\frac{t}{v-c}, \frac{2t}{v-c}\right)$$.

Given the second mover’s strategy above, the first mover will always choose FP since

$$\pi_A(FP, PWYW) = \frac{(t + \lambda(v-c))^2}{8t} > \frac{3\lambda(v-c)}{4} - \frac{\lambda^2(v-c)^2}{4t} = \pi_A(PWYW, FP)$$

in the range $$\lambda \in \left(\frac{t}{v-c}, \frac{2t}{v-c}\right)$$ and

$$\pi_A(FP, FP) = \frac{t}{2} \geq \frac{3\lambda(v-c)}{4} - \frac{\lambda^2(v-c)^2}{4t} = \pi_A(PWYW, FP)$$

otherwise.

Substituting $$v = kc/2$$ into the bounds of $$\lambda \in \left(\frac{t}{v-c}, \frac{2t}{v-c}\right)$$ yields the resulting inequality in Proposition 3.

**D.4 Proposition 4**

When the monopolist chooses FP, consumer surplus $$CS$$ of consumers who buy at price $$p = \frac{c(k+1)}{2}$$ is:

$$CS_{FP} = \frac{1}{kc} \int_{p}^{kc} (u-p)du \bigg|_{p=\frac{c(k+1)}{2}} = \frac{c(k-1)^2}{8k}.$$
Under PWYW, consumer surplus consists of the free-riders’ surplus plus the surplus of fair buyers whose consumption utility exceeds $c$:

$$CS_{PWYW} = \frac{\theta}{kc} \int_{p=0}^{kc} (u-p) du + \frac{1-\theta}{kc} \int_{c}^{kc} (u-c - \lambda(u-c)) du$$

$$= \frac{\theta kc}{2} + \frac{(1-\theta)c(k-1)^2(1-\lambda)}{2k}.$$ 

Hence $CS_{PWYW,FP} > CS_{FP,FP}$ if and only if

$$\lambda < \bar{\lambda} \approx \frac{(k-1)^2(3-4\theta) + 4k^2\theta}{4(1-\theta)(k-1)^2}.$$ 

For PWYW to be weakly preferred by both buyers and seller, there must be some values of $\lambda$ such that $\hat{\lambda} \leq \lambda \leq \bar{\lambda}$. This requires that

$$\bar{\lambda} = \frac{(k-1)^2(3-4\theta) + 4k^2\theta}{4(1-\theta)(k-1)^2} \geq \frac{(k-1)^2 + 4\theta k}{2(1-\theta)(k-1)^2} = \hat{\lambda},$$

that is,

$$\theta \leq \frac{(k-1)^2}{4}.$$ 

### D.5 Proposition 5

It is straightforward to calculate consumer surplus in both the Bertrand FP-pooling equilibrium and the separating equilibrium. In the Bertrand equilibrium (FP,FP), total consumer surplus is:

$$CS_{(FP,FP)} = \frac{c(k-1)^2}{2k}.$$ 

In the separating equilibrium, the total consumer welfare of the free-riders, the fair buyers with low $u_i$ buying from the PWYW seller, and the fair buyers with high $u_i$
buying from the FP seller can be expressed as

\[ CS_{(FP, PWYW)} = \frac{\theta kc}{2} + \frac{(1 - \theta)(4 - 3\lambda)c(k - 1)^2}{8k}. \]

Hence \( CS_{PWYW} > CS_{FP} \) if and only if

\[ \lambda < \lambda^c = \frac{4\theta(2k - 1)}{3(k - 1)^2(1 - \theta)}. \]

Since \( \lambda^c < \lambda^* \) (as given in Proposition 2), it follows that whenever the separating equilibrium obtains, \( \lambda \geq \lambda^* > \lambda^c \) and the consumers prefer the pooling equilibrium.

**D.6 Proposition 6**

In the FP-pooling equilibrium, consumer surplus is

\[ CS_{(FP, FP)} = v - c - \frac{5t}{4}. \]

In the separating equilibrium, consumer surplus is

\[ CS_{(PWYW, FP)} = v - c + \frac{\lambda^2(v - c)^2 - 14\lambda(v - c)t - 7t^2}{16t}. \]

This is always less than \( CS_{(FP, FP)} \) above unless

\[ \lambda \leq \frac{t}{v - c} \quad \text{or} \quad \lambda \geq \frac{13t}{v - c}. \]

We consider only the first region, since restricting the location of the indifferent consumer to \( x \geq 0 \) results in \( \lambda \leq \frac{3t}{v - c} \). Hence, when the separating equilibrium obtains at \( \frac{t}{v - c} < \lambda < \frac{2t}{v - c} \), consumers are never better off.
D.7 Proposition 7

The game tree is solved by backward induction starting at node B.4. When both sellers choose FP, the best response strategy of Seller B is defined by:

\[
 p_B = \begin{cases} 
 \frac{c(k+1)}{2} & \text{if } p_A > \frac{c(k+1)}{2} \Rightarrow \pi_A = 0, \pi_B = \frac{c(k-1)^2}{4k} \\
 p_A - c & \text{if } c < p_A \leq \frac{c(k+1)}{2} \Rightarrow \pi_A = 0, \pi_B \approx \frac{(kc-p_A)(p_A-c)}{kc} \\
 c & \text{if } p_A \leq c \Rightarrow \pi_A \leq 0, \pi_B = 0.
\end{cases}
\]

In all three cases, \( \pi_A \leq 0 \). Therefore at node A.2 Seller A will choose \( p_A \) that gives him positive profit, which is only the case if Seller B chooses PWYW.

If Seller A competes against PWYW, it is clear that setting \( p_A \geq c(\lambda k - \lambda + 1) \), \( u_A \geq kc \) and Seller A will not get any sales. He will also not set \( p_A \leq c \), as this will yield zero or negative profit. These regions are therefore excluded from Seller A’s strategy space in A.2, and his fixed price will instead lie in \( (c, c(\lambda k - \lambda + 1)) \). In this range of \( p_A \), Seller B’s PWYW profit can be expressed as:

\[
\pi_B = \frac{1-\theta}{kc} \int_c^{c+\frac{p_A-c}{k}} \lambda(u-c)du - \theta c \\
= \frac{1-\theta}{2\lambda kc} (p_A-c)^2 - \theta c.
\]

When \( \lambda \leq 1/2 \), Seller B’s profit under PWYW is always less than that under FP.\(^{30}\) When \( \lambda > 1/2 \), Seller B’s PWYW profit will be greater than his FP profit if

\[
\hat{p}_A = c \left[ 1 + \sqrt{\frac{\lambda(k-1)^2 + 4\theta\lambda k}{2(1-\theta)}} \right] < p_A.
\]

\(^{30}\)To see this, note that \( \pi_B \) under FP and PWYW are both increasing in the domain \( (c, c(\lambda k - \lambda + 1)) \). The former is concave, the latter convex. Evaluating \( \pi_B \) under both FP and PWYW at the endpoints \( p_A = c \) and \( p_A = c(\lambda k - \lambda + 1) \) shows that the PWYW profit lies below the FP profit and the result follows.
For $\hat{p}_A < c(\lambda k - \lambda + 1)$, a necessary condition is

$$\lambda > \hat{\lambda} = \frac{(k - 1)^2 + 4\theta k}{2(1 - \theta)(k - 1)^2} > \frac{1}{2}.$$  

In this case,

$$\pi_A = \frac{1 - \theta}{kc} (p_A - c) \left( kc - c - \frac{p_A - c}{\lambda} \right)$$

is decreasing in the domain $p_A \in (\hat{p}_A, c(\lambda k - \lambda + 1))$. Hence, at A.2 Seller A sets

$$p_A = \hat{p}_A + \varepsilon.$$ 

At B.3, given Seller A has chosen PWYW, the optimal fixed price for Seller B also lies in $(c, c(\lambda k - \lambda + 1))$.

$$\pi_B = \frac{1 - \theta}{kc} (p_B - c) \left( kc - c - \frac{p_B - c}{\lambda} \right)$$

is maximized at

$$p_B = \frac{\lambda c (k - 1)}{2} + c,$$

giving

$$\pi_B = \frac{(1 - \theta)\lambda c (k - 1)^2}{4k}, \quad \pi_A = \frac{(1 - \theta)\lambda c (k - 1)^2}{8k} - \theta c.$$  

At B.1, if B chooses PWYW both sellers split the profits and each gets

$$\pi_A = \pi_B = \frac{(1 - \theta)\lambda c (k - 1)^2}{4k} - \frac{\theta c}{2}.$$  

Clearly, with $\theta > 0$ Seller B will choose FP at B.1.

At A.1, given $\lambda \leq \hat{\lambda}$, Seller A choosing FP always results in his competitor also choosing FP. Hence, no matter what fixed price Seller A sets, he always ends up
with zero profit. He therefore chooses PWYW and earns

$$\pi_A = \frac{(1-\theta)\lambda c(k-1)^2}{8k} - \theta c$$

which will be positive as long as

$$\lambda > \lambda^* = \frac{8\theta k}{(1-\theta)(k-1)^2}.$$

When $\lambda > \hat{\lambda}$, Seller A can get positive profit when he chooses FP:

$$\pi_A \approx \frac{1-\theta}{kc} (\hat{p}_A - c) \left( kc - c - \frac{\hat{p}_A - c}{\lambda} \right).$$

This is always greater than his PWYW profit. Hence Seller A will choose FP as long as $\lambda > \hat{\lambda}$.

Therefore, the subgame perfect equilibrium outcomes are as summarized in Proposition 7.

**D.8 Proposition 8**

Consider the end node where both sellers choose FP. Given $p_R$, Seller L’s optimal strategy is to set $p_L^*(p_R) = \frac{p_R + t + c}{2}$. Consequently, profits for both sellers as a function of $p_R$ are:

$$\pi_L = \frac{(p_R + t - c)^2}{8t} \quad \pi_R = \left( \frac{3t - p_R + c}{4t} \right) (p_R - c).$$

Suppose, given $p_R$, that Seller L decides to offer PWYW instead at node B.2.

---

31To see this, set $\lambda = 1$ and $\theta = 0$. At this best case scenario, PWYW profit still lies below FP profit.
Profits for both sellers as a function of $p_R$ are:

$$\pi_L = \lambda(v-c)\left(\frac{t + p_R - c - \lambda(v-c)}{2t}\right) \quad \pi_R = \left(\frac{t - p_R + c + \lambda(v-c)}{2t}\right) (p_R - c).$$

It is straightforward to show that $\pi_L$ is always weakly greater under FP than PWYW for any value of $p_R$. Hence, the second mover Seller L will always choose FP at B.2. Consequently, whenever Seller R chooses FP at A.1, he is guaranteed a profit of $\pi_R = \left(\frac{t - p_R + c + \lambda(v-c)}{2t}\right) (p_R - c)$. Maximising profit with respect to $p_R$, the optimal price is $p_R^* = 3t/2 + c$ and $\pi_R^* = 9t/16$.

If Seller R instead chooses PWYW at A.1, the resulting outcomes are as described in the simultaneous pricing case of Section 5 (see the proof of Proposition 3). The second mover will always find it more profitable to choose FP, as a result profit for the first mover is

$$\pi_R = \lambda(v-c)\left(\frac{3}{4} - \frac{\lambda(v-c)}{4t}\right).$$

This is always less than or equal to $9t/16$, and as a result the first mover Seller R will always choose FP.

**D.9 Proposition 9**

We analyse the PWYW profit for different values of $p_m > c$.

When $c < p_m < c(\lambda k - \lambda + 1)$, profit can be expressed as

$$\pi_m = \frac{\theta}{kc} \int_{p_m}^{kc} (p_m - c)du + \frac{1 - \theta}{kc} \left(\int_{p_m}^{c + \frac{p_m - c}{k}} (p_m - c)du + \int_{c + \frac{p_m - c}{k}}^{kc} (c + \lambda(u - c) - c)du\right)$$

$$= \frac{\theta}{kc} (p_m - c)(kc - p_m)$$

$$+ \frac{1 - \theta}{kc} \left[ (p_m - c)^2 \left(\frac{1}{\lambda} - 1\right) + \frac{\lambda}{2} \left(c^2(k-1)^2 - \frac{(p_m - c)^2}{\lambda^2}\right)\right]. \quad (1)$$
The optimal minimum price is then

\[ p^*_m = c \left( 1 + \frac{\theta \lambda (k-1)}{2\lambda + \theta - 1} \right). \]  

(2)

When \( \lambda > 1/2 \), \( p^*_m \) will lie in the relevant domain and will never exceed \( c(k+1)/2 \), the monopoly fixed price. Otherwise, the maximum profit is attained at \( p_m = c(\lambda k - \lambda + 1) - \varepsilon \).

When \( c(\lambda k - \lambda + 1) \leq p_m \leq kc \), the minimum price will be binding for all fair consumers. Hence, all purchases will be made at \( p_m \) and profit can be expressed as

\[ \pi_m = \frac{1}{kc} (p_m - c)(kc - p_m). \]

Similar to the fixed-price case, this is maximized at \( p_m = c(k+1)/2 \) which will lie in the relevant domain if \( \lambda < 1/2 \). Otherwise, the maximum profit is attained at \( p_m = c(\lambda k - \lambda + 1) \).

Comparing profit levels at these two domains, it follows that when \( \lambda \leq 1/2 \), the maximum profit of \( c(k-1)^2/(4k) \) is attained at \( p_m = c(k+1)/2 \), which is the same as the fixed-price case. From Proposition 1, this is greater than the original PWYW profit (since \( \lambda \leq 1/2 \) and \( \theta = 0 \)). When \( \lambda > 1/2 \), the maximum profit is attained at \( p^*_m \) as given in equation (2). While the expression for profit is complex, it is straightforward to show that this profit exceeds the fixed-price profit of \( c(k-1)^2/(4k) \): when \( \lambda > 1/2 \), \( c(k+1)/2 \) lies in the domain considered for equation (1). Evaluating this expression at \( p_m = c(k+1)/2 \) results in

\[ \pi_m \left( \frac{c(k+1)}{2} \right) = \frac{c(k-1)^2}{4k} \left( 2\theta - 1 + \frac{1 - \theta}{2\lambda} + 2(1 - \theta)\lambda \right) \]

which is greater than \( c(k-1)^2/(4k) \). Since from equation (2) \( p^*_m \) is the optimal solution, it follows then that \( \pi_m(p^*_m) > \pi_m(c(k+1)/2) > c(k-1)^2/(4k) \). Similarly,
\[ \pi_m(p_m^*) > \pi_m(c) = (1 - \theta)\lambda c(k - 1)^2/(2k) \geq (1 - \theta)\lambda c(k - 1)^2/(2k) - \theta c = \pi_{PWYW}. \]

Hence, PWYW with a minimum price weakly dominates fixed-pricing and strictly dominates PWYW without a minimum price.

### D.10 Proposition 10

#### D.10.1 Best response function of PWYW seller facing PWYW seller

We begin by finding Seller B’s optimal response at node B.3, choosing the profit-maximizing minimum price given that Seller A has also chosen PWYW with a minimum price.

i. When \(0 \leq p_A \leq c\), the optimal strategy for Seller B is to set \(p_A < p_B \leq c\). This ensures that all the free-riders go to Seller A, without reducing profit from the fair buyers.

Outside this range of \(p_A\), it is never optimal to set \(p_B < c\) as all free-riders will then go to Seller B, causing a loss. Similarly, it is straightforward to check that it is never optimal to overtake \(p_A\), as any profit (which will only materialize if \(p_B\) is non-binding for at least some fair buyers) will be higher at \(p_B = p_A\).

Hence we proceed by considering best response strategies in the range \([c, p_A]\).

ii. When \(c < p_A < c(\lambda k - \lambda + 1)\), setting \(p_B = c\) gives:

\[
\pi_B(c) = \frac{1 - \theta}{kc} \left( \int_c^{\mu_A} \lambda(u-c)du + \frac{1}{2} \int_{\mu_A}^{kc} \lambda(u-c)du \right) \\
= \frac{(1 - \theta)\lambda}{4kc} \left( c^2(k-1)^2 + \frac{(p_A-c)^2}{\lambda^2} \right). 
\tag{3}
\]

At \(p_A = p_B\), Seller A gets exactly half of the free-riders, half the fair buyers paying the minimum price, and half the fair buyers paying PWYW price. His
profit is
\[
\pi_B(p_A) = \frac{\theta}{2kc}(p_A - c)(kc - p_A) + \frac{1 - \theta}{2kc} \left( \int_{p_A}^{u_A} (p_A - c) du + \int_{u_A}^{kc} \lambda(u - c) du \right)
\]
\[
= \frac{\theta}{2kc}(p_A - c)(kc - p_A) + \frac{1 - \theta}{2kc}(p_A - c)^2 \left( \frac{1}{\lambda} - 1 \right)
\]
\[
+ \frac{(1 - \theta)\lambda}{4kc} \left( c^2(k - 1)^2 - \frac{(p_A - c)^2}{\lambda^2} \right). \tag{4}
\]

If Seller B sets \(c < p_B < p_A\), his profit comes from the free-riders, the fair buyers with \(p_B \leq u_i < u_B\) paying the minimum price, fair buyers with \(u_B \leq u_i < u_A\) paying the PWYW price \(p_i\), and half the fair buyers with \(u_i \geq u_A\) who randomize:
\[
\pi_B(p_B) = \frac{\theta}{kc}(p_B - c)(kc - p_B)
\]
\[
+ \frac{1 - \theta}{kc} \left( \int_{p_B}^{u_B} (p_B - c) du + \int_{u_B}^{u_A} \lambda(u - c) du + \frac{1}{2} \int_{u_A}^{kc} \lambda(u - c) du \right)
\]
\[
= \frac{\theta}{kc}(p_B - c)(kc - p_B) + \frac{1 - \theta}{kc}(p_B - c)^2 \left( \frac{1}{\lambda} - 1 \right)
\]
\[
+ \frac{1 - \theta}{2\lambda kc}(p_A^2 - p_B^2 - 2c(p_A - p_B))
\]
\[
+ \frac{(1 - \theta)\lambda}{4kc} \left( c^2(k - 1)^2 - \frac{(p_A - c)^2}{\lambda^2} \right). \tag{5}
\]

Combining equations (3)-(5), \(\pi_B\) is continuous at \(p_B = c\) but jumps down at \(p_B = p_A\). Solving the first-order condition yields the same optimal value \(p_B^*\) as in equation (2), which will lie in the relevant domain only if \(\lambda > 1/2(> (1 - \theta)/2)\). In this case, \(\pi_B(p_B^*)\) is the maximum profit. Otherwise, the profit function (5) is increasing in the domain and the optimal strategy is to set \(p_B = p_A - \epsilon\).

iii. When \(p_A = c(\lambda k - \lambda + 1)\), setting \(p_B = c\) gives no profit from free-riders but
Seller B captures the whole fair buyers market.

\[
\pi_B(c) = \frac{1 - \theta}{kc} \int_c^{kc} \lambda(u - c)du = \frac{(1 - \theta)\lambda c(k - 1)^2}{2k}.
\]

At \(p_A = c(\lambda k - \lambda + 1)\), the minimum price is binding for all buyers and the two sellers share the market.

\[
\pi_B(c(\lambda k - \lambda + 1)) = \frac{\lambda c(1 - \lambda)(k - 1)^2}{2k}.
\]

If Seller A sets \(c < p_A < c(\lambda k - \lambda + 1)\),

\[
\pi_B(p_B) = \frac{\theta}{kc} (p_B - c)(kc - p_B) + \frac{1 - \theta}{kc} \left( \int_{p_B}^{u_B} (p_B - c)du + \int_{u_B}^{kc} \lambda(u - c)du \right)
\]

\[
= \frac{\theta}{kc} (p_B - c)(kc - p_B) + \frac{1 - \theta}{kc} (p_B - c)^2 \left( \frac{1}{\lambda} - 1 \right)
\]

\[
+ \frac{(1 - \theta)\lambda}{2kc} \left( c^2(k - 1)^2 - \frac{(p_B - c)^2}{\lambda^2} \right).
\]  (6)

Note again that \(\pi_B\) is continuous at \(p_B = c\) but jumps down at \(p_B = p_A\). The first-order-condition is again solved by \(p_B^*\) which will lie in the relevant domain if \(\lambda > 1/2\). In this case, \(\pi_B(p_B^*)\) is the maximum profit. Otherwise, the profit function (6) is increasing in the domain and the optimal strategy is to set \(p_B = c(\lambda k - \lambda + 1) - \varepsilon\).

iv. When \(c(\lambda k - \lambda + 1) < p_A \leq kc\), setting \(p_B = c\) again gives Seller B the whole market:

\[
\pi_B(c) = \frac{1 - \theta}{kc} \int_c^{kc} \lambda(u - c)du = \frac{(1 - \theta)\lambda c(k - 1)^2}{2k}.
\]

At \(c(\lambda k - \lambda + 1) \leq p_B < p_A\), Seller B will get the whole market of both free-
riders and fair buyers who will all pay the minimum \( p_A \):

\[
\pi_B(p_B) = \frac{1}{kc}(p_B - c)(kc - p_B).
\]

In this region, the optimal \( p_B \) is the monopoly price \( c(k + 1)/2 \) if \( \lambda < 1/2 \) and \( p_A > c(k + 1)/2 \). If \( \lambda \geq 1/2 \), Seller B should go as low as possible and set \( p_B = c(\lambda k - \lambda + 1) \), while if \( c(k + 1)/2 \geq p_A \) then Seller B should go as high as possible and set \( p_B = p_A - \varepsilon \). Setting \( p_B = p_A \) results in the two sellers sharing the market equally, with all buyers randomizing their choice of sellers and paying the same price at both stores:

\[
\pi_B(p_A) = \frac{1}{2kc}(p_A - c)(kc - p_A).
\]

To get some of the fair buyers to share their surplus, Seller B needs to set \( c < p_B < c(\lambda k - \lambda + 1) \). In this domain,

\[
\pi_B(p_B) = \frac{\theta}{kc}(p_B - c)(kc - p_B) + \frac{1 - \theta}{kc}(\int_{p_B}^{u_B}(p_B - c)du + \int_{u_B}^{kc}\lambda(u - c)du)
\]

\[
= \frac{\theta}{kc}(p_B - c)(kc - p_B) + \frac{1 - \theta}{kc}(p_B - c)^2 \left( \frac{1}{\lambda} - 1 \right) + \frac{(1 - \theta)\lambda}{2kc} \left( c^2(k - 1)^2 - \frac{(p_B - c)^2}{\lambda^2} \right). \tag{7}
\]

Note that \( \pi_B \) is continuous at both \( p_B = c \) and \( p_B = c(\lambda k - \lambda + 1) \). Solving the first-order condition again yields \( p_B^* \) which will lie in the relevant domain if \( \lambda > 1/2 \). In this case, \( \pi_B(p_B^*) \) is the maximum profit. Otherwise, the profit function \( (7) \) is increasing in the domain and the optimal strategy is to set \( p_A = c(\lambda k - \lambda + 1) - \varepsilon \).
Hence, Seller B’s best response strategy can be summarized below:

\[ 0 \leq p_A \leq c \implies p_A < p_B \leq c \]

\[ c < p_A < c(\lambda k - \lambda + 1) \implies p_B = \begin{cases} p_A, & \text{if } \lambda > \frac{1}{2} \text{ and } p_A > p_B^* \\ p_B = p_A - \epsilon, & \text{otherwise} \end{cases} \]

\[ c(\lambda k - \lambda + 1) \leq p_A \leq kc \implies p_B = \begin{cases} p_B^*, & \text{if } \lambda > \frac{1}{2} \\ \frac{c(k+1)}{2}, & \text{if } \lambda \leq \frac{1}{2} \text{ and } p_A > \frac{c(k+1)}{2} \\ p_A - \epsilon, & \text{if } \lambda \leq \frac{1}{2} \text{ and } p_A \leq \frac{c(k+1)}{2}. \end{cases} \]

**D.10.2 Best response function of FP seller facing PWYW seller**

At B.4, Seller B has to find the profit-maximizing fixed price given that Seller A has chosen PWYW and a minimum price. Straight away, we can exclude any \( p_B \leq c \) from Seller B’s best response strategy since this gives negative or zero profit. However, note that this time it is possible for Seller B to set a fixed price that is higher than his competitor’s minimum price, because for buyers with a high enough valuation (\( u_i > u_B \)) it is cheaper to go to Seller B and pay the fixed price than go to Seller A and share his (high) surplus.

i. When \( p_A \leq c \), setting \( c < p_B < c(\lambda k - \lambda + 1) \) means that Seller B gets all the buyers with valuation \( u_i > u_B \) and profit is

\[ \pi_B(p_B) = \frac{1 - \theta}{kc} (p_B - c)(kc - u_B) = \frac{1 - \theta}{kc} (p_B - c) \left( kc - c - \frac{p_B - c}{\lambda} \right). \]
This expression is maximized at
\[ p_B^* = c \left( 1 + \frac{\lambda(k-1)}{2} \right) \]  
(8)
and at this point profit equals
\[ \pi_B(p_B^*) = \frac{(1-\theta)\lambda c(k-1)^2}{4k} \].

Setting \( c(\lambda k - \lambda + 1) \leq p_B < kc \) means that \( u_B \geq kc \) and there is no buyer for which \( u_i > u_B \). Hence profit is zero.

ii. When \( c < p_A < c(\lambda k - \lambda + 1) \), setting \( c < p_B < p_A \). Seller B gets the whole market and profit is
\[ \pi_B(p_B) = \frac{1}{kc} (p_B - c)(kc - p_B) \]  
(9)
which is maximized at \( p_B = c(k + 1)/2 \). Matching Seller A’s price, \( p_B = p_A \), they share the market for free-riders and fair buyers with valuations \( p_B < u_i < u_B \), but Seller B gets all the fair buyers with higher valuations. Profit is
\[
\pi_B(p_A) = \frac{\theta}{2kc} (p_A - c)(kc - p_A) + \frac{1-\theta}{2kc} (p_A - c)^2 \left( \frac{1}{\lambda} - 1 \right) \\
+ \frac{1-\theta}{kc} (p_A - c) \left( kc - c - \frac{p_A - c}{\lambda} \right).
\]

If Seller B sets a higher price than Seller A, such that \( p_A < p_B < c(1 + \lambda) \), he gets all the buyers with \( u_i > u_B \) and profit is
\[
\pi_B(p_B) = \frac{1-\theta}{kc} (p_B - c)(kc - u_B) = \frac{1-\theta}{kc} (p_B - c) \left( kc - c - \frac{p_B - c}{\lambda} \right) \]  
(10)

which is also optimized at $p_B^*$ as given in equation (8). Profit as given in equation (9) is continuous at $p_B = c$ but jumps down at $p_B = p_A$, and again to the function given in equation (10). It will be maximized at $c(k+1)/2$ if this value is less than $p_A$. However, if $p_A \leq c(k+1)/2$, we need to consider the other potential maximum point $p_B^*$. If it lies outside its profit-maximizing range, i.e., if $c(1+\lambda(k-1)/2) \leq p_A$, the best strategy is to set $p_B = p_A - \varepsilon$. Otherwise, we need to compare profits at $p_B = p_A - \varepsilon$ and $p_B = p_B^*$ to see which is higher. Set $p_B = p_A - \varepsilon$ if

$$
\pi_B(p_A - \varepsilon) \approx \frac{1}{kc}(p_A - c)(kc - p_A) > \frac{(1 - \theta)\lambda c(k-1)^2}{4k} = \pi_B(p_B^*)
$$

$$
\implies p_A > \frac{c(k+1) - c(k-1)\sqrt{1-\lambda + \lambda\theta}}{2} := p.
$$

Otherwise, set $p_B = p_B^*$. Note that $p < p_B^*$.

iii. When $c(\lambda k - \lambda + 1) \leq p_A \leq kc$, Seller B can only get a share of the market by setting a lower price than $p_A$, in which case he gets the whole market. Profit is

$$
\pi_B(p_B) = \frac{1}{kc}(p_B - c)(kc - p_B)
$$

which is maximized at $c(k+1)/2$. If $c(k+1)/2 \geq p_A$, Seller B should undercut his competitor and set $p_B = p_A - \varepsilon$.

Hence, Seller B’s profit-maximizing fixed price can be summarized below:

$$
0 \leq p_A \leq c \implies p_B = p_B^*.
$$
\[ c < p_A < c(\lambda k - \lambda + 1) \implies \begin{cases} 
  p_B = p_B^*, & \text{if } p_A \leq p \\
  p_B = p_A - \epsilon, & \text{if } p < p_A \leq \frac{c(k+1)}{2} \\
  p_B = \frac{c(k+1)}{2}, & \text{if } p_A > \frac{c(k+1)}{2} 
\end{cases} \]

\[ c(\lambda k - \lambda + 1) \leq p_A \leq kc \implies \begin{cases} 
  p_B = \frac{c(k+1)}{2}, & \text{if } p_A > \frac{c(k+1)}{2} \\
  p_B = p_A - \epsilon, & \text{if } p_A \leq \frac{c(k+1)}{2}. 
\end{cases} \]

### D.10.3 Best response function of PWYW seller facing FP seller

We proceed by first assuming that Seller A sets a fixed price \( p_A \), and finding the profit-maximizing minimum price for Seller B. Note first that Seller A will never set \( p_A < c \) since this gives negative profit, and these values are therefore excluded from Seller A’s strategy set.

Firstly, note that Seller B will never set a minimum price that is higher than Seller A’s fixed price. If he did, all the free-riders and fair buyers with \( u_i < u_B \) paying the minimum price would go to Seller A with the lower fixed price, and the fair buyers with higher valuations who would have paid \( p_i \) will also go to Seller A since \( u_i \geq u_B > u_A \) and hence \( p_A < p_i \).

i. When \( p_A = c \), setting \( p_B \leq c \) is not binding for the fair buyers, but in the interest of reducing loss from free-riders it is optimal to set \( p_B = c = p_A \). However, then \( u_B = u_A = c \) and all fair buyers with \( u_i \geq p_A \) would rather go to the fixed-price seller. Hence Seller B is indifferent between setting \( p_B = c \) or \( p_B > c \), in both cases \( \pi_B = 0 \).
ii. When \( c < p_A < c(\lambda k - \lambda + 1) \), it is not optimal to set \( p_B < c \) since Seller B can always reduce the loss due to free-riders by setting \( p_B = c \). He will capture all the free-riders and fair buyers with \( u_i < u_A \), and his profit is

\[
\pi_B(c) = \frac{1 - \theta}{kc} \int_c^{u_A} \lambda(u - c)du = \frac{(1 - \theta)(p_A - c)^2}{2\lambda ck}.
\]  

(11)

Setting \( p_B = p_A \) means that Seller B gets half the free-riders and half the fair buyers with \( u_i < u_B = u_A \), all of whom pay \( p_B \). His profit is

\[
\pi_B(p_A) = \frac{1}{2} \left( \frac{\theta}{kc} (p_A - c)(kc - p_A) + \frac{1 - \theta}{kc} (p_A - c)(u_A - p_A) \right).
\]  

(12)

Setting \( c < p_B < p_A \) means that Seller B captures all the free-riders and fair buyers with \( u_i < u_B = u_A \), and his profit is

\[
\pi_B(p_B) = \frac{\theta}{kc} (p_B - c)(kc - p_B) + \frac{1 - \theta}{kc} \left( \int_{p_B}^{u_B} (p_B - c)du + \int_{u_B}^{u_A} \lambda(u - c)du \right)
\]
\[
= \frac{\theta}{kc} (p_B - c)(kc - p_B) + \frac{1 - \theta}{kc} (p_B - c)^2 \left( \frac{1}{\lambda} - 1 \right)
\]
\[
+ \frac{1 - \theta}{2\lambda ck} (p_A^2 - p_B^2 - 2c(p_A - p_B)).
\]  

(13)

Combining equations (11)-(13), \( \pi_B \) is continuous at \( p_B = c \) but jumps down at \( p_B = p_A \). Solving the first-order condition yields the same optimal value \( p_B^* \) as in equation (2), which will lie in the relevant domain only if \( \lambda > 1/2 \). In this case, \( \pi_B(p_B^*) \) is the maximum profit. Otherwise, the profit function (13) is increasing in the domain and the optimal strategy is to set \( p_B = p_A - \epsilon \).

iii. When \( p_A = c(\lambda k - \lambda + 1) \), setting \( p_B = c \) means that Seller B captures the
whole fair buyers market.

\[ \pi_B(c) = \frac{1 - \theta}{kc} \int_c^{kc} \lambda(u - c)du = \frac{(1 - \theta)\lambda c(k - 1)^2}{2k}. \]

At \( p_B = c(\lambda k - \lambda + 1) \), the minimum price is binding for all buyers and the two sellers share the market.

\[ \pi_B(c(\lambda k - \lambda + 1)) = \frac{\lambda c(1 - \lambda)(k - 1)^2}{2k}. \]

If Seller B sets \( c < p_B < c(\lambda k - \lambda + 1) \),

\[
\begin{align*}
\pi_B(p_B) &= \frac{\theta}{kc} (p_B - c)(kc - p_B) + \frac{1 - \theta}{kc} \left( \int_{u_B}^{p_B} (p_B - c)du + \int_{u_B}^{kc} \lambda(u - c)du \right) \\
&= \frac{\theta}{kc} (p_B - c)(kc - p_B) + \frac{1 - \theta}{kc} (p_B - c)^2 \left( \frac{1}{\lambda} - 1 \right) \\
&+ \frac{(1 - \theta)\lambda}{2kc} \left( c^2(k - 1)^2 - \frac{(p_B - c)^2}{\lambda^2} \right). 
\end{align*}
\]

(14)

Note again that \( \pi_B \) is continuous at \( p_B = c \) but jumps down at \( p_B = p_A \). The first-order-condition is again solved by \( p_B^* \) which will lie in the relevant domain if \( \lambda > 1/2 \). In this case, \( \pi_B(p_B^*) \) is the maximum profit. Otherwise, the profit function (14) is increasing in the domain and the optimal strategy is to set \( p_B = c(\lambda k - \lambda + 1) - \epsilon \).

iv. When \( c(\lambda k - \lambda + 1) < p_A \leq kc \), at \( p_B = c \), Seller B will again get the whole market of fair buyers paying PWYW, together with the free-riders who do not generate any profit:

\[ \pi_B(c) = \frac{1 - \theta}{kc} \int_c^{kc} \lambda(u - c)du = \frac{(1 - \theta)\lambda c(k - 1)^2}{2k}. \]

At \( c(\lambda k - \lambda + 1) \leq p_B < p_A \), Seller B will get the whole market of both free-
riders and fair buyers who will all pay the minimum \( p_B \):

\[
\pi_B(p_B) = \frac{1}{k} (p_B - c)(k - p_B).
\]

In this region, the optimal \( p_B \) is \( c(k + 1)/2 \) if \( \lambda < 1/2 \) and \( p_A > c(k + 1)/2 \). If \( \lambda \geq 1/2 \), Seller B should go as low as possible and set \( p_B = c(\lambda k - \lambda + 1) \), while if \( c(k + 1)/2 \geq p_A \) then Seller B should go as high as possible and set \( p_B = p_A - \epsilon \). Setting \( p_B = p_A \) results in the two sellers sharing the market equally, with all buyers randomizing their choice of sellers and paying the same price at both stores:

\[
\pi_B(p_A) = \frac{1}{2k} (p_A - c)(k - p_A).
\]

To get some of the fair buyers to share their surplus, Seller B needs to set \( c < p_B < c(\lambda k - \lambda + 1) \). In this domain,

\[
\pi_B(p_B) = \frac{\theta}{k} (p_B - c)(k - p_B) + \frac{1 - \theta}{k} \left( \int_{p_B}^{u_B} (p_B - c)du + \int_{u_B}^{kc} \lambda(u - c)du \right)
\]

\[
= \frac{\theta}{k} (p_B - c)(k - p_B) + \frac{1 - \theta}{k} (p_B - c)^2 \left( \frac{1}{\lambda} - 1 \right)
\]

\[
+ \frac{1 - \theta}{2k} \lambda \left( c^2 - \frac{(p_B - c)^2}{\lambda^2} \right).
\]

Note that \( \pi_B \) is continuous at both \( p_B = c \) and \( p_B = c(\lambda k - \lambda + 1) \). Solving the first-order condition again yields \( p_B^* \) which will lie in the relevant domain if \( \lambda > 1/2 \). In this case, \( \pi_B(p_B^*) \) is the maximum profit. Otherwise, the profit function (15) is increasing in the domain and the optimal strategy is to set \( p_B = c(\lambda k - \lambda + 1) - \epsilon \).
Hence,

\[ p_A = c \implies p_B \in [c, kc]. \]

\[ c < p_A < c(\lambda k - \lambda + 1) \implies \begin{cases} 
    p_B = p_B^*, & \text{if } \lambda > \frac{1}{2} \text{ and } p_A > p_B^* \\
    p_B = p_A - \varepsilon, & \text{otherwise.}
\end{cases} \]

\[ c(\lambda k - \lambda + 1) \leq p_A \leq kc \implies p_B = \begin{cases} 
    p_B^*, & \text{if } \lambda > \frac{1}{2} \\
    \frac{c(k+1)}{2}, & \text{if } \lambda \leq \frac{1}{2} \text{ and } p_A > \frac{c(k+1)}{2} \\
    p_A - \varepsilon, & \text{if } \lambda \leq \frac{1}{2} \text{ and } p_A \leq \frac{c(k+1)}{2}.
\end{cases} \]

From the best response functions given above, we can find the equilibrium prices set by each firm in the end nodes of the simultaneous competition as depicted in Figure 10.

In the (FP,FP) Bertrand equilibrium, both sellers set \( p = c \) and makes zero profit. In the identical (PWYW,FP) and (FP,PWYW) end nodes, there is no Nash equilibrium in pure strategies. In the (PWYW,PWYW) end node, the only Nash equilibrium is both sellers setting \( (c, c) \). The case where \( \lambda > 1/2 \) is plotted in Figure 13, where the only intersection of the best response functions is found at \( (c, c) \). The other cases are straightforward to draw, giving the results stated in Proposition 10.
Figure 13: Best response functions for end node (PWYW,PWYW)
E Robustness Checks

E.1 Endogenous consumer choice

Suppose that each consumer maximises his utility by choosing whether or not to free-ride, in contrast with the model presented in Section 3 where his type (free-rider or fair) is determined exogenously. We show here that the qualitative results obtained in Propositions 1 and 2 are unchanged. To do so, we introduce a social or moral penalty for free-riding. For example, in this analysis we use the function $U_i = (1 - \lambda)(1 - \frac{1}{r})u_i$, $r > 1$, for the free-rider’s utility: his consumption utility of the good is now discounted by a factor $(1 - \lambda)$ to match the fair consumers’ marginal utility, and further by the factor $(1 - \frac{1}{r})$ to account for the penalty. The higher $r$ is, the lower the social penalty, and hence the higher the free-rider’s residual utility.

Hence, facing a PWYW seller, the typical consumer will choose to free-ride up until $u_i = rc$, at which point he is better off paying a fair price $c + \lambda(u_i - c)$. The consumer’s utility when he is a free-rider, fair consumer, or chooses to pay a fixed price under competition is illustrated in Figure 14.

With the above consumer preference, the PWYW monopolist profit is now

$$\pi_{PWYW} = \frac{1}{kc} \int_{0}^{rc} \frac{1}{kc} (-c)du + \frac{1}{kc} \int_{rc}^{kc} \lambda(u - c)du$$

$$= - \frac{rc}{k} + \frac{\lambda c(k - r)(k + r - 2)}{2k}$$

which will be greater than the monopolist FP profit only if

$$\lambda > \frac{(k - 1)^2 + 4r}{2(k - r)(k + r - 2)}.$$  

Note that other free-rider utility functions can be substituted here, as long as the indifferent consumer has consumption utility $u_i = rc$. An example is where the free-rider is penalised by the amount $u_i - (1 - \lambda)c(r - 1)$, such that his utility is constant at $U_i = (1 - \lambda)c(r - 1)$.  

\[32\text{Note that other free-rider utility functions can be substituted here, as long as the indifferent consumer has consumption utility } u_i = rc. \text{ An example is where the free-rider is penalised by the amount } u_i - (1 - \lambda)c(r - 1), \text{ such that his utility is constant at } U_i = (1 - \lambda)c(r - 1).\]
Figure 14: Consumer utility functions, $\lambda=0.6$

\[ U_i^{PWYW} \text{(free-rider)} = (1 - \lambda)(1 - \frac{1}{r})u_i \]
\[ U_i^{PWYW} \text{(fair)} = (1 - \lambda)(u_i - c) \]
\[ U_i^{FF} = u_i - p \]
That is, when the level of surplus-sharing is sufficiently high as per Proposition 1.

We now turn to the competition setting described in Section 4. When both sellers choose PWYW, they split the monopolist PWYW profit and each earns

$$\pi_A = \pi_B = -\frac{rc}{2k} + \frac{\lambda c(k-r)(k+r-2)}{4k}.$$  

When both choose FP, each seller earns zero profit under Bertrand competition. When one seller chooses PWYW and the other FP, the FP seller captures all consumers with $u_i \geq u_p$ as given in Section 4, while the PWYW seller captures those with $u_i < u_p$, some of whom will free-ride ($u_i < rc$). For the existence of fair consumers who choose the PWYW seller, we impose the restriction $r < \frac{k+1}{2}$. The FP seller’s profit-maximising price is identical to that given in Section 4:

$$p^* = c \left(1 + \frac{\lambda(k-1)}{2}\right)$$

and his profit is

$$\pi_{FP} = \frac{\lambda c(k-1)^2}{4k}.$$ 

The PWYW seller’s profit is thus

$$\pi_{PWYW} = \frac{1}{kc} \int_0^{rc} (-c)du + \frac{1}{kc} \int_{rc}^{u_p} \lambda(u-c)du$$

$$= -\frac{rc}{k} + \frac{\lambda}{kc} \left[ \frac{(p-c)^2}{2\lambda^2} - \frac{c^2(r-1)^2}{2} \right]$$

$$= -\frac{rc}{k} + \frac{\lambda c}{k} \left[ \frac{(k-1)^2}{8} - \frac{(r-1)^2}{2} \right].$$

The sellers’ profits at the various end nodes are summarised in Figure[15].

When the first mover has chosen PWYW, it is straightforward to show that the
second mover will always choose FP:

$$\frac{-rc}{2k} + \frac{\lambda c(k-r)(k+r-2)}{4k} < \frac{\lambda c(k-1)^2}{4k}.$$  

On the other hand, when the first mover has chosen FP, the second mover will choose PWYW only if the PWYW profit exceeds zero, that is if

$$\lambda > \frac{8r}{(k-1)^2 - 4(r-1)^2}.$$  

Consequently, the first mover will always choose FP in the first stage, as per the result stated in Proposition 2.

### E.2 Heterogeneous surplus-sharing

Suppose that consumers share their surplus according to the following PWYW price:

$$p_i = \lambda_i u_i.$$  

$\lambda_i$ represents each consumer's individual surplus-sharing proportion, which depends on his own degree of social preferences. His PWYW price increases the
more he cares about social preferences and the higher his consumption utility from the good. Assume that $\lambda_i$ is independent and identically distributed according to some continuous distribution with expected value $\lambda$. While we drop the free-riding parameter $\theta$, the presence of free-riders is captured in this new model by assuming the presence of consumers with $\lambda_i = 0$. As the proportion of such consumers increases, the expected value of the surplus-sharing parameter $\lambda$ naturally decreases.

The PWYW monopolist’s expected profit is thus given by

$$E\pi_{PWYW} = E \int_0^{kc} \frac{1}{kc} (\lambda_i u_i - c) du = \frac{\lambda kc}{2} - c$$

which, similar to Proposition 1, is only greater than the monopolist FP profit when

$$\lambda > \frac{(k + 1)^2}{2k^2}.$$ 

Under the homogeneous goods competition setting in Section 4, at the end node when both sellers choose PWYW each earns half the monopolist profit above. When one seller chooses PWYW and the other FP, given $\lambda_i$, the buyer will choose PWYW whenever $u_i < u_p$ and FP when $u_i > u_p$ where

$$u_p = \frac{p}{\lambda_i}.$$ 

The FP seller’s expected demand is thus positive as long as $Eu_p > kc$. Given his profit-maximising price of

$$p = \frac{\lambda kc + c}{2},$$

positive expected demand translates to the condition that $\lambda > 1/k$ which we assume will be satisfied for the rest of the analysis.\footnote{If $\lambda \leq 1/k$, demand for the FP seller is zero, resulting in $E\pi_{FP} = 0$. The PWYW seller captures the whole market and gets the PWYW monopolist expected profit, which in this case is $\leq 0$. The} Accordingly, the FP seller’s expected
profit is
\[ \mathbf{E}\pi_{FP} = \frac{c(\lambda k - 1)^2}{4\lambda k} \]
and his competitor earns
\[ \mathbf{E}\pi_{PWYW} = \frac{c(\lambda k + 1)(\lambda k - 3)}{8\lambda k}. \]

It is straightforward to derive the subgame perfect Nash equilibrium, which is identical to Proposition 2 except for the fact that the threshold value is \( \lambda^* = 3/k \).

Suppose now that we have the differentiated goods setting as per Section 5 where again \( v = kc/2 \). With two PWYW sellers in the market, each seller gets half the monopolist PWYW expected profit of \( (\lambda v - c)/2 \). When Seller L on the left chooses PWYW and Seller R chooses FP, the indifferent consumer is now located at \( x = (t + p_R - \lambda_i v)/(2t) \). Consequently, Seller R sets \( p = (t + c + \lambda v)/2 \) and each seller earns
\[ \mathbf{E}\pi_{FP} = \frac{(t + \lambda v - c)^2}{8t} \quad \mathbf{E}\pi_{PWYW} = \frac{3(\lambda v - c)}{4} - \frac{(\lambda v - c)^2}{4t}. \]

Assuming the indifferent seller chooses FP then yields the same result as Proposition 3, but with threshold values
\[ \frac{t + c}{v} < \lambda < \frac{2t + c}{v} \]
for the separating equilibrium to obtain.

resulting subgame perfect equilibrium is thus FP-pooling.
E.3 Product differentiation with free-riders

Consider the setting of Section 5, however we now assume that the proportion of free-riders $\theta > 0$. When both firms choose FP, profits are unaffected by free-riders:

$$\pi_i = \frac{t}{2}.$$  

When both firms choose PWYW, they are both negatively affected by free-riders:

$$\pi_i = \frac{(1 - \theta)\lambda(v - c) - \theta c}{2}.$$  

When Seller L chooses PWYW and Seller R FP, the indifferent fair consumer is still located at $x = \frac{(t + p_R - c - \lambda(v - c))/(2t)}{x}$ but the free-rider will be closer to Seller R as more of them will pay the transport cost to take the good for free: $x = \frac{(t + p_R)/(2t)}{x}$. With this demand structure, the profit-maximizing fixed-price seller now optimally sets

$$p_R = \frac{t + c + (1 - \theta)(c + \lambda(v - c))}{2}$$

which is lower than in the case of no free-riders, to attract some of the free-riders as well. Hence the location of both indifferent fair consumer and free-riders decrease (or become closer to the PWYW seller) as $\theta$ increases. Profits are

$$\pi_L = (1 - \theta)\lambda(v - c) \left( \frac{3t + c - (1 + \theta)(c + \lambda(v - c))}{4t} \right)$$

$$- \theta c \left( \frac{3t + c + (1 - \theta)(c + \lambda(v - c))}{4t} \right)$$

while

$$\pi_R = \frac{(t - c + (1 - \theta)(c + \lambda(v - c))^2}{8t}.$$  

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both of which are decreasing in $\theta$.

The equilibrium analysis follows as per the proof of Proposition 3. Suppose the first mover A has chosen PWYW. The second mover B will always choose FP since

$$\pi_B(PWYW, PWYW) < \pi_B(PWYW, FP).$$

On the other hand, if the first mover has chosen FP, the second mover will choose PWYW if

$$\pi_B(FP, PWYW) > \pi_B(FP, FP).$$

The above inequality is less likely to hold compared to the case of no free-riders, as the left hand side is decreasing in $\theta$ while the right hand side is independent of $\theta$.

Given the second mover’s strategy above, the first mover will always choose FP since

$$\pi_A(FP, PWYW) > \pi_A(PWYW, FP)$$

given $\pi_B(FP, PWYW) > \pi_B(FP, FP)$ holds.\(^{34}\) Otherwise,

$$\pi_A(FP, FP) > \pi_A(PWYW, FP)$$

as the same inequality holds for Seller B.

\(^{34}\)To prove this, set $\theta = 0$. At this best case scenario, the PWYW profit is still lower than the FP profit.