Assigning Refugees to Landlords in Sweden: Efficient Stable Maximum Matchings*

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Abstract

The member states of the European Union received 1.2 million first time asylum applications in 2015 (a doubling compared to 2014). Even if asylum will be granted for many of the refugees that made the journey to Europe, several obstacles for successful integration remain. This paper focuses on one of these obstacles, namely the problem of finding housing for refugees once they have been granted asylum. In particular, the focus is restricted to the situation in Sweden during 2015–2016 and it is demonstrated that market design can play an important role in a partial solution to the problem. More specifically, because almost all accommodation options are exhausted in Sweden, the paper investigates a matching system, closely related to the system adopted by the European NGO "Refugees Welcome", and proposes an easy-to-implement mechanism that finds an efficient stable maximum matching. Such matching guarantees that housing is efficiently provided to a maximum number of refugees and that no refugee prefers some landlord to their current match when, at the same time, that specific landlord prefers that refugee to his current match.

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1 Introduction

The European refugee crisis began in 2015 when a rising number of refugees made the journey to Europe to seek asylum. The member states of the European Union received 1.2 million first time asylum applications (more than a doubling compared to 2014).[1] Apart from the Dublin Regulation, which dictates that the member state in which an asylum seeker enters first is obliged to render asylum, there has been no systematic way to divide refugees between the member states. Obviously, this puts great pressure on member states located at the external border of the European Union, and more specifically, on Greece, Hungary and Italy.

In an attempt to reduce pressure on these three member states, the European Commission decided in September 2015 on a temporary European relocation scheme for 120,000 refugees who are in need of international protection.[2] The relocation scheme did, however, not specify which refugees should be relocated to which member states. This specific problem has attracted interest among researchers and more systematic ways to relocate refugees between European Union member states have been proposed. For example, Jones and Teytelboym (2016a) propose a system where member states and refugees submit their preferences about which refugees they most wish to host and which state they most wish to be hosted by, respectively, to a centralized clearing house which matches member states and refugees according to these preferences.

Even if membership quotas are settled and a centralized matching relocation system is in place, several obstacles for successful integration remain. This paper focuses on one of these obstacles, namely the problem of finding housing for refugees once they have been relocated to a European Union membership state, and, in particular, how market design can play an important role in the solution to the problem. The background to the housing problem will be described from the perspective of the situation in Sweden during 2015–2016.

In 2015, the population of Sweden was 9.9 million which accounted for around 1.4 percent of the population in Europe. Yet, 12.4 percent of the asylum seekers in the European Union in 2015 were registered in Sweden which made Sweden the state in the European Union with most asylum seekers per capita.[3] A refugee who enters Sweden is temporarily placed at a Migration Board accommodation facility in anticipation of either a deportation order or a permanent residence permit. The average waiting time for this decision was 15 months in May 2016.[4] Refugees who are granted permanent residence permits are, under Swedish law, entitled to a number of establishment measures (e.g., accommodation and a monthly allowance), and their legal status is upgraded from "asylum seekers" to "refugees with a permanent residence permit".[5] The local

[1]Eurostat News, Release 44/2016, March 4, 2016.

[2]Eurpoean Commission, Statement 15/5697, September 22, 2015.

[3]Eurostat News, Release 44/2016, March 4, 2016.

[4]Swedish Migration Board, www.migrationsverket.se/Kontakta-oss/Tid-till-beslut.html, May 13, 2016.

[5]The Swedish terminology for "refugees with a permanent residence permit" is "nyanländ" but this terminology will, for convenience, be slightly abused in the remaining part of the paper as the word "refugees" will be used instead of the correct terminology "refugees with a permanent residence permit".

municipality where the refugee is registered has the responsibility to find appropriate accommodation. In this process, the refugee must leave the Migration Board accommodation facility since the legal responsibility for the refugee is transferred from the state to the local municipality.

One problem in Sweden is that almost all accommodation options are exhausted. In March 2015, it was estimated that 9,300 persons with a permanent residence permit still lived in a Migration Board accommodation facility and that, at least, 14,100 residential units were needed before the end of 2016 just to accommodate those who are granted a residence permit.[6] This estimation was updated in February 2016 to at least 20,000 new residential units only in the spring of 2016 provided that there is no drastic increase in the number of asylum seekers.[7] These facts, together with a new legislation, effective from March 1, 2016, stating that all municipalities have to accept refugees puts even more pressure on some municipalities to find additional residential units. This has forced some municipalities to consider extraordinary actions. One example is the passenger ship Ocean Gala leased for use as an asylum accommodation with room for nearly 800 people in Utansjö port outside the city Härnösand in the north east of Sweden.[8] Another example is a temporary tent camp with a capacity to accommodate 1,520 asylum seekers that was scheduled to open in December 2015 on Revingehed armor training ground 20 kilometers east of the city of Lund in the south of Sweden.[9] Hence, it is urgent to find residential units for refugees, not only because they are entitled to it under Swedish law, but also because they are blocking asylum seekers from accommodation at Migration Board accommodation facilities.

A key observation, and a possible solution to the above described problem, can be found in a report from "The Swedish National Board of Housing, Building and Planning" in 2013, where it is estimated that 90 percent of the general housing shortage in Sweden can be explained by inefficient use of the existing housing stock.[10] More precisely, due to rent control, tenants tend to live in apartments which are too big for their circumstances. The question is then how this situation can be utilized. The answer may be found in a recent survey that concluded that 31 percent of the Swedish households are willing to accommodate refugees in their homes.[11] Of course, a stated willingness to accommodate a refugee and actually accommodating a refugee are two different things, and it should also be noted that the general view on refugees in Sweden was not as positive in the spring of 2017 as it was in the fall of 2015.[12] However, there were 4,766,000

[6] "Nyanländas boendesituation – delrapport", The Swedish National Board of Housing, Building and Planning, Rapport 2015:10.

[7] "More than 20 000 new places needed in accommodation in the spring", Swedish Migration Board, February 19, 2016.

[8] "Migrationsverket visste inte att miljonbåten var på väg", June 15, 2016, SVT.

[9] "Första asylsökande har flyttat in i tältlägret i Revinge", December 10, 2015, Aftonbladet.

[10] "Bostadsbristen och hyressättningssystemet – ett kunskapsunderlag", Marknadsrapport, The Swedish National Board of Housing, Building and Planning, 2013.

[11] "Svenska folkets attityder till flyktingar", September 24, 2015, DN/Ipsos.

[12] "Allmänhetens uppfattning om invandringen", March 25, 2016, Demoskop.

households in Sweden in January 2015[13] and if only 1 percent of the households (instead of 30 percent) are willing to accommodate a refugee, there are still 47,660 households willing to host refugees. Hence, to release the pressure on municipalities to find housing for refugees, voluntarily supplied private housing can be utilized.[14] In this way, beds that are occupied by refugees with a permanent residence permit at the Migration Board accommodation facilities can be used for asylum seekers.

Language	*Native speakers*	*Language*	*Native speakers*	*Language*	*Native speakers*
Swedish	8,000,000	Polish	76,000	Norwegian	54,000
Finnish	200,000	Spanish	75,000	English	54,000
Arabic	155,000	Persian	74,000	Somali	53,000
BCSM	130,000	German	72,000	Armenian	52,000
Kurdish	84,000	Danish	57,000	Turkish	45,000

Table 1: The 15 most common native languages in Sweden in 2012 (estimated). The population of Sweden was 9,556,000 in 2012. BCSM is an abbreviation for Bosnian, Croatian, Slovenian and Macedonian. Source: Parkvall (2016).

In several meetings at various levels in the Swedish administration, e.g., with the State Secretary to the Minister of Housing and the Swedish Migration Board, the authors of this paper presented a version of the theoretical matching model described in this paper. The model contains a set of "landlords" (i.e., private persons) with capacity and willingness to accommodate refugees in their private homes and a set of refugee families with permanent residence permits. In the model, a refugee family and a landlord find each other mutually acceptable if they have a spoken language in common and if the number of family members does not exceed the capacity of the landlord. The communication requirement is key and its importance has been stressed by politicians in, e.g., the above mentioned meetings. It is also a requirement in, e.g., the non-centralized system adopted by the European NGO "Refugees Welcome" to match refugees with private persons. It is then natural to ask: is it even possible to find private landlords that are able to communicate with refugees? Table 1 states the 15 most common native languages in Sweden in 2012 and provides a partial answer to the question. As can be seen from the table, Arabic is the third most common native language and all Kurdish, Persian and Somali qualify for the list. These languages are spoken by more than 50 percent of the asylum seekers in 2015 and 2016. Given this and the observation that the number of native language speakers gives a lower bound for how many persons speak the language, shows that the communication requirement is not unreasonable.

[13]"Antal hushåll i Sverige", 2016, Statistics Sweden.

[14]In fact, many Swedish municipalities are today actively searching for private persons that are willing to accommodate refugees in their private homes even if private persons and refugee families are matched in a non-centralized way. Examples of such municipalities include Stockholms stad, Lunds kommun, Ängleholms kommun, Nynäshamns kommun, Kristianstads kommun, Nacka kommun, Botkyrka kommun, Håbo kommun, Härryda kommun, and Lerums kommun.

In many market design applications, it is realistic to assume that agents can form a ranking over potential matches.[15] In, for example, the school choice problem, parents have access to information about schools in their locality and can, based on this information, form preferences over schools. In the considered refugee matching application, however, it may be difficult for landlords (refugee families) to provide preferences as there are thousands of refugee families (landlords) in the system and it is difficult to gather complete information about all these families (landlords) and even if such information is available, it is not clear how to process it. For this reason, preferences will, like in the standard kidney exchange problem (Roth et al., 2004), be induced from reported data (in the standard kidney exchange problem, patient preferences over donors are typically induced based on medical data such as tissue type antibodies and blood group).

Preferences will be induced in two steps. First, landlords classify refugee families to belong to different indifference classes and have strict preferences over the indifference classes. This classification is based on mutual acceptability together with a few natural assumptions related to, e.g., monotonicity in family size (see Assumptions 1–4). Second, preferences for refugee families are based only on mutual acceptability. This type of preference structure on two-sided matching markets was recently studied by Haeringer and Iehlé (2017). They demonstrated, using data from the junior academic job market for French mathematicians, that the assumption that preferences are based on mutual acceptability on one side of the market (in this case, the refugee side) can be made almost without loss of generality whenever the interest is directed towards stable matchings since this simpler preference structure is a good approximation of a strict preference ordering. Hence, there is also an empirical motivation for the simple type of preference structure considered in this paper.

Given the above type of induced preferences, our objective is to find a mechanism for assigning refugee families to private landlords. In particular, interest is directed towards mechanisms that selects efficient, stable and maximum matchings. The stability axiom means that no refugee family strictly prefers some landlord to being unmatched when, at the same time, that specific landlord strictly prefers that refugee family to his current match. Consequently, stability in the considered setting guarantees a lower welfare bound for the participating private landlords as the landlords can be made assure that if they are matched to some refugee family, there is at least no unmatched refugee family that they strictly prefer to their current match. It is also well-known that unstable mechanisms tend to die out while stable mechanisms survive the test of time (Roth, 2008). Maximality means that a maximum number of refugees are matched to landlords or, equivalently, that a maximum number of privately supplied beds are utilized. This axiom can be motivated by the above described acute shortage of residential units in Sweden. Our main results

[15] Examples include school choice (Balinski and Sönmez, 1999; Abdulkadiroğlu and Sönmez, 2003), entry-level job markets (Roth and Peranson, 1999), course allocation (Budish and Cantillon, 2012), kidney exchange (Roth et al., 2004) and cadet-branch matching (Sönmez and Switzer, 2013; Sönmez, 2013). For an overview of the matching and market design literature see, e.g., Roth and Sotomayor (1990) or Sönmez and Ünver (2011).

show the existence of efficient stable maximum matchings. This is surprising as in many of the existing applications, even two of the three requirements-efficiency, stability and maximum-are incompatible.

Even if a variety of problems have been investigated in different market design contexts, almost no attention has been directed towards problems related to refugee assignment. There are, however, a few papers on the local refugee matching problem, i.e., the problem of finding out where in a country that refugees should be settled once they have been granted protection. Jones and Teytelboym (2016b) describe in general terms how a two-sided matching system can be constructed when assigning refugees to localities and detail how this system can be applied in order to meet the British government's commitment to resettle 20,000 Syrian refugees. Delacretaz et al. (2016) consider a two-sided matching market for the local refugee match and propose three different refugee resettlement systems that can be used by hosting countries under different circumstances.[16] The proposed solutions are all based on different versions of the Deferred Acceptance Algorithm and the Top Trading Cycles Algorithm, and these mechanisms cannot be used to solve the allocation problem considered in this paper. Moreover, none of the matching systems in the above mentioned papers focuses on efficient stable maximum matchings.

The remaining part of the paper is outlined as follows. Section 2 introduces the refugee assignment problem and the basic ingredients of the matching model. Results related to efficient, stable and maximum matchings as well as to manipulability and non-manipulability are stated in Section 3. Some concluding remarks are provided in Section 4. All proofs are relegated to the Appendix.

2 The Model and Basic Definitions

2.1 The Refugee Assignment Problem

Each refugee family contains a number of family members that wish to be accommodated by a landlord. The set of refugee families is given by $I = \{1, \ldots, |I|\}$ and the vector $q_I = (q_1, \ldots, q_{|I|})$ specifies the size q_i of each refugee family $i \in I$. For convenience, the term "refugee" will often be used instead of "refugee family" and it is then understood that the refugee is part of a refugee family with a specific number of family members. Moreover, the expression "capacity of refugee i" will often be used to indicate the number of family members in refugee family i.

Landlords are private persons supplying voluntarily parts of their homes to refugee families. Exactly how many refugees a landlord can accommodate is determined by his capacity. The set of landlords is given by $C = \{c_1, \ldots, c_{|C|}\}$ and the vector $q_C = (q_{c_1}, \ldots, q_{c_{|C|}})$ specifies the capacity q_c of each landlord $c \in C$. Landlords and refugee families speak at least one language.

[16] A recent paper by Aziz et al. (2017) investigates various notions of stability in the refugee assignment model introduced by Delacretaz et al. (2016).

The set L contains the languages spoken by the refugee families and the landlords in $I \cup C$. The languages spoken by refugee family $i \in I$ are collected in the non-empty set $L(i) \subseteq L$. Landlords have strict preferences over languages they speak. Formally, a list of strict preferences $\succeq = (\succeq_{c_1}, \ldots, \succeq_{c_{|C|}})$ specifies the preferences $\succeq_c$ over $L \cup \{c\}$ for each landlord $c \in C$. It will be sometimes convenient not to separate refugees from landlords. In this case, we refer to agent v who belongs to the set $V = C \cup I$.

2.2 Induced Preferences

Language l is acceptable for landlord c if $l \succ_c c$. The set of acceptable languages for landlord c is denoted by $A(\succeq_c)$. Both language and capacity constraints play an important role in determining which refugees are acceptable for landlords and vice versa. More precisely, let $l_c(i) = \max_{\succeq_c} L(i) \cup \{\emptyset\}$ denote the most preferred spoken language of refugee i from the perspective of landlord c. Then refugee i is acceptable for landlord c if and only if the most preferred language spoken by refugee i from the perspective of landlord c is acceptable for landlord c and if the size of refugee i does not exceed the capacity of landlord c, i.e., if and only if $l_c(i) \in A(\succeq_c)$ and $q_i \leq q_c$. By symmetry, landlord c is acceptable for refugee i if and only if refugee i is acceptable for landlord c. An agent who is not acceptable is unacceptable. The following assumptions will be maintained for the remaining part of this paper.

Assumption 1. If a landlord accommodates a refugee family, then the landlord has to accommodate all members of the family.

Assumption 2. Landlords can only accommodate acceptable refugee families and refugee families can only be accommodated by acceptable landlords.

Assumption 3. Landlords can accommodate at most one refugee family.

Assumption 4. Landlords strictly prefer a larger refugee family to a smaller refugee family if both refugee families are acceptable.

The first assumption captures the humanitarian requirement that refugee families should be kept intact and, in addition, the legal requirement (specified in the Dublin Regulation) that refugee family members should not be separated. Assumption 2 has two important implications. First, participating landlords (refugee families) should not face the risk of being forced to accommodate refugee families (being accommodated by landlords) that they find unacceptable. Second, landlords should be able to communicate with the refugee families they accommodate (recall the discussion from the Introduction). Assumption 3 takes care of a potential conflict with the Swedish tax law. More precisely, if a landlord accommodates more than one family, the Swedish Tax Agency may classify the landlord's house as a "hotel"[17] meaning that the landlord formally

[17]See www.skatteverket.se.

has to operate a hotel business and, consequently, has to follow the regulations associated with this type of enterprise, pay taxes accordingly, etc.. The study of such enterprises is beyond the scope of this paper and is excluded by Assumption 3. Finally, Assumption 4 captures the idea that landlords receive a family size dependent monetary compensation for accommodating refugee families (this type of monotonic compensation schemes exist today, for example in the city of Stockholm[18]). Given that landlords only offer unused parts of their homes, it is realistic to assume that larger refugee families are strictly preferred to smaller ones.

Remark 1. The basic refugee assignment model considered here is a two-sided matching model with capacities as in, e.g., Abdulkadiroğlu and Sönmez (2003), Abdulkadiroğlu et al. (2009), Balinski and Sönmez (1999) and Gale and Shapley (1962), where the number of offered beds and the number of needed beds are the capacities of the landlords and the refugee families, respectively. It is, however, not a many-to-many matching model (Echenique and Oviedo, 2006; Konishi and Ünver, 2006a) even though landlords offer several beds and refugee families may need multiple beds. This follows from Assumption 3, since agents are matched to at most one agent from the other side of the market. □

Given the notion of acceptability and the above four assumptions, it is possible to derive an induced preference profile R for the agents in V. Let R_c denote the induced preference relation R_c for landlord $c \in C$ over the set $I \cup \{c\}$. Let also P_c and I_c denote the strict and the indifference part of the preference relation R_c, respectively. The induced preference relation R_c for landlord c is described by:

- cP_ci if and only if refugee i is unacceptable,
- iP_cc if and only if refugee i is acceptable,
- iP_cj if refugees $i, j \in I$ are acceptable and $q_i > q_j$,
- iP_cj if refugees $i, j \in I$ are acceptable, $q_i = q_j$ and $l_c(i) \succ_c l_c(j)$,
- iI_cj if refugees $i, j \in I$ are acceptable, $q_i = q_j$ and $l_c(i) = l_c(j)$.

Similarly as above, let R_i denote the induced preference relation R_i for refugee $i \in I$ over the set $C \cup \{i\}$, and let P_i and I_i denote its strict and indifference relations, respectively. The induced preference relation R_i for refugee i is based only on mutual acceptability (see the discussion related to Haeringer and Iehlé, 2017, from the Introduction):

- iP_ic if and only if landlord c is unacceptable,
- cP_ii if and only if landlord c is acceptable,

[18] See www.stockholm.se/-/Nyheter/Nyanlanda/Hyr-ut-din-bostad.

- cI_ic' if landlords $c, c' \in C$ are acceptable.

Let $R = (R_v)_{v \in V}$ denote the induced (preference) profile for the agents in V. The set of all such profiles is denoted by $\mathcal{P}^V$. A profile $R \in \mathcal{P}^V$ may also be written as (R_v, R_{-v}) when the preference relation R_v of agent $v \in V$ is of particular importance.

2.3 Matchings and Mechanisms

Landlords (refugees) are either unmatched or matched to a refugee (to a landlord) under the restriction that a landlord $c \in C$ is matched to refugee $i \in I$ if and only if refugee i is matched to landlord c. Formally, a matching is a function $\mu : C \cup I \to C \cup I$ such that $\mu(c) \in I \cup \{c\}$ for all $c \in C$, $\mu(i) \in C \cup \{i\}$ for all $i \in I$, and $\mu(c) = i$ if and only if $\mu(i) = c$. Agent v is unmatched at matching μ if $\mu(v) = v$. Given a matching μ, the matched landlords and the matched refugees are collected in the sets $\mu(C) \equiv \{c \in C : \mu(c) \neq c\}$ and $\mu(I) \equiv \{i \in I : \mu(i) \neq i\}$, respectively. A matching μ is *feasible* at profile $R \in \mathcal{P}^V$ if $\mu(v)R_v v$ for all $v \in V$, i.e., if each agent is matched to an acceptable agent or remains unmatched. The set of all feasible matchings at profile $R \in \mathcal{P}^V$ is denoted by $\mathcal{A}(R)$.

Let $|\mu| = \sum_{i \in \mu(I)} q_i = \sum_{c \in \mu(C)} q_{\mu(c)}$ denote the cardinality of matching μ, i.e., the total number of matched refugee family members at matching μ. A matching $\mu \in \mathcal{A}(R)$ is *maximum* at profile $R \in \mathcal{P}^V$ if there exists no other matching $\mu' \in \mathcal{A}(R)$ such that $|\mu'| > |\mu|$. A matching $\mu \in \mathcal{A}(R)$ is *stable* at profile $R \in \mathcal{P}^V$ if there is no blocking pair, i.e., if there exist no landlord-refugee pair (c, i) such that $iP_c\mu(c)$ and $cP_i\mu(i)$. Note that given the induced preferences considered in this paper, stability means that no refugee family strictly prefers some landlord to being unmatched when, at the same time, that specific landlord strictly prefers that refugee family to his current match. A matching $\mu \in \mathcal{A}(R)$ is *(Pareto) efficient* at profile $R \in \mathcal{P}^V$ if there exists no other matching $\mu' \in \mathcal{A}(R)$ such that $\mu'(v)R_v\mu(v)$ for all $v \in V$ and $\mu'(v)P_v\mu(v)$ for some $v \in V$. An *efficient stable maximum* matching is a matching which is efficient, stable and maximum. All efficient stable maximum matchings at profile $R \in \mathcal{P}^V$ are gathered in the set $\mathcal{X}(R)$.

A *(matching) mechanism* is a function $f : \mathcal{P}^V \to \cup_{R \in \mathcal{P}^V} \mathcal{A}(R)$ choosing a feasible matching $f(R) \in \mathcal{A}(R)$ for any profile $R \in \mathcal{P}^V$. Let $f_v(R)$ denote the match for agent v at matching $f(R)$. A mechanism f is manipulable by agent $v \in V$ at profile $R \in \mathcal{P}^V$ if there is a profile $(R'_v, R_{-v}) \in \mathcal{P}^V$ such that $f_v(R'_v, R_{-v})P_v f_v(R)$. A mechanism f which is not manipulable by any agent $v \in V$, at any profile $R \in \mathcal{P}^V$, is non-manipulable.[19] A mechanism f which makes a selection from the set $\mathcal{X}(R)$ at any profile $R \in \mathcal{P}^V$ is an *efficient stable maximum mechanism*.

[19]This paper will not consider the possibility for groups of agents to manipulate a mechanism. See Barberà et al. (2016) for a recent paper on the relation between individual manipulability and group manipulability.

3 Results

3.1 Efficient Stable Maximum Matchings

Given the induced preferences and the interest in efficient stable maximum matchings, it is first established that none of the three axioms of interest are implied by any of the other two axioms (e.g., that it not generally is that case that a stable maximum matching is efficient) on the domain $\mathcal{P}^V$.[20]

Proposition 1. (i) An efficient stable matching is not necessarily maximum,

(ii) a stable maximum matching is not necessarily efficient, and

(iii) an efficient maximum matching is not necessarily stable.

The next result establishes that an efficient stable maximum matching exists for any profile $R \in \mathcal{P}^V$. The underlying reason is that preferences over languages are correlated in the following sense; if a landlord finds a certain language acceptable, then the landlord is indifferent between any two refugee families of the same size that speak this language, and any two refugee families speaking the same language are indifferent between any two acceptable landlords speaking their language. In Remark 2 in the Appendix we also show that stable maximum matchings do not necessarily exist when either (i) refugees are allowed to express strict preferences between acceptable landlords or (ii) landlords do not necessarily prefer larger acceptable families to smaller ones.

Theorem 1. For any profile $R \in \mathcal{P}^V$, there exists an efficient stable maximum matching.

The existence of an efficient stable maximum mechanism f on the domain $\mathcal{P}^V$ is implied by Theorem 1. It will next be demonstrated that such a mechanism can be formulated as a maximum weight matching problem. This technique is, in similarity with the Deferred Acceptance Algorithm (Gale and Shapley, 1962) and the Top Trading Cycles Mechanism (Shapley and Scarf, 1974), frequently adopted in the market design literature to solve various matching problems. For example, solution methods based on a maximum weight matching problem has recently been applied in problems related to kidney exchange (Biró et al., 2009), teacher assignment (Combey et al., 2016) and school choice (Kesten and Ünver, 2015; Biró and Gudmundsson, 2017).

To formulate the maximum weight matching problem for a given refugee assignment problem, a bipartite graph needs to be defined and specific values must be attached to the edges in the graph. Because the structure of the graph is dependent on the given profile $R \in \mathcal{P}^V$, the bipartite graph is described by $g = (V, E, R)$ where V is a set of vertices and E a set of edges. Every vertex in the graph g corresponds to an agent in $V = C \cup I$. Moreover, there is an edge between

[20] Part (ii) of Proposition 1 was first established by Gale and Shapley (1962). It is restated here for completeness.

landlord $c \in C$ and refugee $i \in I$, denoted by ci, if and only if they find each other mutually acceptable at profile R. Let $E(g)$ denote the set of edges in graph g. Note also that any matching $\mu \in \mathcal{A}(R)$ can be described as a subset of edges $E(\mu) \subseteq E(g)$ for the graph $g = (V, E, R)$, i.e., if landlord c is matched to refugee i at matching μ, then $ci \in E(\mu)$.

A weighted graph (g, w) is defined by a graph $g = (V, E, R)$ and a vector of edge weights $w \equiv (w_{ci})_{ci \in E(g)}$ where w_{ci} is the weight assigned to edge $ci \in E(g)$. Let $S(g, w, \mu) = \sum_{ci \in E(\mu)} w_{ci}$ be the sum of all edge weights at matching $\mu = E(\mu)$. A matching μ is a maximum weight matching in the weighted graph (g, w) if $S(g, w, \mu) \geq S(g, w, \mu')$ for all matchings $\mu' \in \mathcal{A}(R)$. Let $\mathcal{V}(g, w) \subseteq \mathcal{A}(R)$ denote the set of all maximum weight matchings in the weighted graph (g, w).

To identify a maximum weight matching, the vector of edge weights $w \equiv (w_{ci})_{ci \in E(g)}$ for the weighted graph (g, w) needs to be specified. Let, for this purpose, $|L_c|$ be the number of languages spoken by landlord c. Let also o be the maximal number of languages spoken by any landlord in C, i.e., $o \equiv \max_{c \in C}\{|L_c|\}$. To simultaneously obtain stability and efficiency, it will be important to keep track of how landlords rank the languages of the refugees that they potentially are matched to. To achieve this, let $r_c(\succeq_c, l)$ denote the rank of any language l spoken by landlord c (for example, if landlord c only speaks Arabic and English but prefers the former to the latter according to $\succeq_c$, then Arabic is ranked as 1 and English is ranked as 2). Let now each weight w_{ci} in the vector of edge weights $w \equiv (w_{ci})_{ci \in E(g)}$ be defined by:

$$w_{ci} = |C| o q_i + (1 + |L_c| - r_c(\succeq_c, l_c(i))). \tag{1}$$

The expression $|C|oq_i$ represents the part of the edge weight that is associated to family size whereas the expression $(1 + |L_c| - r_c(\succeq_c, l_c(i)))$ is the part of the edge weight related to how landlord c ranks the most preferred language spoken by refugee i among all languages spoken by landlord c. The former expression is much larger compared to the latter and this difference in size will guarantee the selection of a maximum matching. The latter expression will assure the selection of a stable matching. The two expressions will jointly guarantee efficiency.

Theorem 2. A mechanism f that for each profile $R \in \mathcal{P}^V$ selects a matching from the set $\mathcal{V}(g, w)$ where $g = (V, E, R)$ and the vector of edge weights w is defined by equation (1) is an efficient stable maximum mechanism.

Note that a maximum weight matching can be identified in polynomial time by adopting the Hungarian method of Kuhn (1955) and Munkres (1957).

3.2 Manipulability and Non-Manipulability

It is well-known that there exists no matching mechanism in two-sided matching markets which always selects a stable matching and at the same time gives the agents on both sides of the market incentives to truthfully report their preferences (Roth, 1982). However, a famous result (again by

Roth, 1982) states that a mechanism selecting always a stable matching which is "optimal" for the agents on one side of the market (i.e., the male-optimal or the female-optimal stable matching) will, in general, not give the agents on that side of the market any incentives to misrepresent their preferences. This paper provides two results with a similar flavor for efficient stable maximum mechanisms. More specifically, manipulation by means of language misrepresentation is impossible for refugee families but possible for landlords. Moreover, capacity manipulation[21] is possible for refugee families but not for landlords. To formalize these results, two clarifications are needed. First, it has not been detailed exactly how conflicts between agents are solved in the case when the set $\mathcal{X}(R)$ contains multiple matchings for a given profile $R \in \mathcal{P}^V$ and when agents are not indifferent between all matchings in this set. Second, as hinted above, the meaning of manipulability is not clear as agents report information about both languages *and* capacities.

Suppose now that the set $\mathcal{X}(R)$ contains several matchings at a given profile $R \in \mathcal{P}^V$ and, in addition, that not all agents are indifferent between all matchings in $\mathcal{X}(R)$. In this case, the mechanism must make a specific selection from the set $\mathcal{X}(R)$ based on some type of exogenously given rule. This rule can, for example, be based on a lottery that is conducted before the agents report their preferences or an exogenously given priority order. The important feature is that the rule consistently breaks ties in a way which is independent of the information reported by the agents (if the tie-braking rule is endogenously dependent on reported information, further manipulation possibilities may arise). To formalize, suppose that $f(R) = \mu$ and that matchings μ and μ' belong to $\mathcal{X}(R) \cap \mathcal{X}(R')$ for some profiles $R, R' \in \mathcal{P}^V$, then it cannot be the case that $f(R') = \mu'$. That is, if the mechanism f selects matching μ over matching μ' at some profile, then it cannot be the case that the very same mechanism selects matching μ' over matching μ at some other profile whenever both matchings are efficient, stable and maximum at both these profiles. A mechanism that respects this condition is said to consistently break ties.[22] This is equivalent to requiring that the mechanism uses an priority order over all matchings and for any profile chooses the efficient stable maximum matching which is highest on this priority order.

A first observation is that it is reasonable to assume that family size can be verified by the authorities. However, if such verification is impossible, refugee families may misrepresent their capacity (i.e., family size) in two different ways. First, a family of six members may, for example, claim that they are two separate families with, say, three members each. This type of manipulation will never be successful by Assumption 2 since refugee families prefer not to be matched to any landlord rather than splitting the family. A second type of capacity manipulation may occur if two separate refugee families merge and pretend to be one family (i.e., some type of group manipulation). Such manipulation generally cannot be avoided if interest is directed towards maximum matchings. Just imagine a situation where the landlord with maximal

[21]For more on capacity manipulation, see, e.g., Kesten (2012) or Konishi and Ünver (2006b).

[22]It is not very difficult to define a mechanism f that consistently break ties and for each profile $R \in \mathcal{P}^V$ selects a matching from the set $\mathcal{V}(g, w)$. This can be achieved by adding a "sufficiently small" weight to condition (1) where the additional weight captures the idea in the tie-breaking rule (e.g., agent-based tie-breaking).

capacity can host six refugees and the unique largest refugee family has five members. Then if two unmatched families of, say, three members each merge and claim to be one family with six members, they will necessarily be matched to the landlord with maximal capacity. Given these conclusions, the remaining part of the analysis related to refugee manipulation focuses on language manipulation, i.e., misrepresentation of the set of spoken languages $L(i)$. For this type of manipulation, the following positive result can be obtained.

Proposition 2. Let f be an efficient stable maximum mechanism breaking ties consistently. Then no refugee family $i \in I$ can manipulate f by misrepresenting $L(i)$ at any profile $R \in \mathcal{P}^V$.

Landlords report information related to capacities and languages even if landlords have more degrees of freedom to manipulate by language misrepresentation compared to refugee families since landlords also report a strict ranking $\succeq_c$ over the languages they speak. As it turns out, this additional degree of freedom makes it possible for landlords to manipulate any efficient stable maximum mechanism (in fact, the result is more general than this as it is obvious from the proof of Proposition 3(i) that the result holds for any maximum matching mechanism). A positive result is, however, obtained in terms of capacity manipulation. More precisely, landlords do not have the above described type of possibility to merge with other landlords as it is reasonable to assume that landlords must state their property name, address, etc. when signing up to the centralized matching system. Hence, the only type of capacity manipulation that remains for the landlords is to misrepresent q_c, i.e., the number of available beds. Such manipulation attempts are, however, fruitless as revealed below.

Proposition 3. Let f be an efficient stable maximum matching mechanism breaking ties consistently. Then:

(i) it is generally impossible to prevent landlords from manipulating the mechanism f by misrepresenting preferences over languages $\succeq_c$; but

(ii) no landlord $c \in C$ can manipulate f by misrepresenting q_c at any profile $R \in \mathcal{P}^V$.

4 Concluding Remarks

This paper is one of the first to investigate a matching model related to refugee resettlement and refugee assignment. In fact, we are only aware of a few other matching papers that have investigated this specific problem and all of them have been cited in the Introduction. The point of departure has been the European refugee crisis during 2015–2016 and, more specifically, the acute problem to find housing for refugees in Sweden. Even if the presented matching model is stylized, in the sense that landlords and refugees only are allowed to submit limited information related to ability to communicate (i.e., language) and capacity (i.e., number of available/needed beds), the model is relevant from a policy perspective because the suggested mechanism is easy-to-implement and it can be adopted as is without any modifications. The mechanism can therefore be seen as a first emergency measure to release pressure on the municipalities in their attempts to find additional residential units. Even if it has not been discussed in the paper, one can also imagine that preferences are induced based on other criteria than communication (e.g., geographical preferences) or a combination of several variables (see, e.g., Delacretaz et al., 2016). Hence, this paper should be seen as a first step to solve an acute problem but future research is needed to find alternative proposals. It is, however, clear that the investigated problem is on the highest political agenda in all member states of the European Union and it is therefore crucial that the market design community continues to investigate problems related to refugee matching and refugee resettlement.

Even if this paper has focused exclusively on a very specific refugee matching problem, is has contributed to the matching and market design literature in a broader sense. First, the paper has provided a framework for analysing two-sided matching markets when preferences are incomplete on one side of the market (in this case, the refugee side) and therefore needs to be approximated using the concept of mutual acceptability. Such a market has recently been analyzed by Haeringer and Iehlé (2017) even if their objective is very different from ours, namely to deduce information on stable matchings from partial observation of preferences. Second, it has been demonstrated that a new class of positive non-manipulability results can be obtained on two-sided matching markets even if agents are allowed to report information in *two* dimensions. Third, the model can be seen as an extension of the matching model with a dichotomous domain that was popularized by Bogomolnaia and Moulin (2004) since agents on one side of the market are allowed to provide preferences over different indifference classes of agents on the other side or the market (note that all results presented in this paper hold for such preference structure even if preferences not are induced). The results of this paper then demonstrate that there is no conflict between efficiency, stability, and maximality on a larger domain than the dichotomous one.

Appendix: Proofs

Proof of Proposition 1. The proposition is proved using a simple example. Let $C = \{c_1, c_2, c_3, c_4\}$ and $I = \{1, 2, 3, 4, 5\}$, and suppose that $q_v = 1$ for all $v \in V$. The induced preference profile R is given by the table below (it is straightforward to verify that there exists lists $(\succeq_{c_1}, \dots, \succeq_{c_4})$

and $(L(1),\ldots,L(5))$ which are consistent with profile R)[23].

R_{c_1}	R_{c_2}	R_{c_3}	R_{c_4}	R_1	R_2	R_3	R_4	R_5
1	2	4	2	c_1, c_3	c_2, c_4	c_1, c_2	c_1, c_2, c_3	c_1
4	3	1	c_4					
5	4	c_3						
3	c_2							
c_1								

Consider next the following three matchings:

$$\mu = \begin{pmatrix} c_1 & c_2 & c_3 & c_4 \\ 1 & 2 & 4 & c_4 \end{pmatrix}, \mu' = \begin{pmatrix} c_1 & c_2 & c_3 & c_4 \\ 4 & 3 & 1 & 2 \end{pmatrix}, \mu'' = \begin{pmatrix} c_1 & c_2 & c_3 & c_4 \\ 5 & 3 & 1 & 2 \end{pmatrix}.$$

The interpretation of matching μ is that landlord c_1 is matched to refugee 1, landlord c_2 is matched to refugee 2, landlord c_3 is matched to refugee 4, and that landlord c_4 as well as refugees 3 and 5 are unmatched.

In showing (a) it is easy to verify that μ is stable and efficient, but μ is not maximum because $|\mu| = 3 < 4 = |\mu'|$ and μ' is feasible. In showing (b) it is easy to verify that μ' is stable and maximum, but μ' is not efficient because μ' can be Pareto improved if landlords c_1 and c_3 swap refugees. In showing (c), it is easy to verify that μ'' is efficient and maximum, but μ'' is not stable because $(c_3, 4)$ is a blocking pair of μ''. $\square$

Remark 2. *When either (i) refugees are not indifferent between acceptable landlords or (ii) landlords do not necessarily prefer larger acceptable families to smaller ones, then stable maximum matchings do not necessarily exist.*

Regarding (i), in the above example it is easy to verify that for any maximum matching $\hat{\mu}$ we have $\hat{\mu}(2) = c_4$. But now any such matching contains the blocking pair $(c_2, 2)$ if refugee 2 is allowed to express the strict preference $c_2 P_2 c_4 P_2 2$ over acceptable landlords.

Regarding (ii), in the above example, suppose that refugee 5 has capacity 2. Then for any maximum matching $\hat{\mu}$ we have $\hat{\mu}(5) = c_1$. Now if landlord c_1 is allowed to strictly prefer refugees 1 and 4 (with capacity 1) over refugee 5 with capacity 2, then (if $\hat{\mu}$ were stable), $\hat{\mu}(1) \neq 1$ and $\hat{\mu}(4) \neq 4$. But then we must have $\hat{\mu}(1) = c_3$ and $\hat{\mu}(4) = c_2$. But then $\hat{\mu}(3) = 3$ and $\hat{\mu}$ contains the blocking pair $(c_2, 3)$.

To prove Theorem 1, it will be necessary to introduce tie-breaking. Note that this notion of tie-breaking is different from consistent tie-breaking previously discussed in Section 3. This should cause no confusion as the type of tie-breaking introduced next only is used to prove Theorem 1. Let now $\mathcal{R}^V$ denote the set of all profiles including those which do not necessarily respect language constraints. A profile $R \in \mathcal{R}^V$ is strict if R_v is strict for all $v \in V$. Consider next

[23] Simply let $L = \{l_1, \ldots, l_5\}$ and $L(i) = \{l_i\}$ for all $i \in I$. Then, for instance, $l_4 \succ_{c_3} l_1 \succ_{c_3} \succ_{c_3} c_3$.

the profiles $R^t, R \in \mathcal{R}^V$ and matching μ. A profile R^t breaks ties in R if and only if (i) for all landlords $c \in C$ and all distinct agents $v, v' \in I \cup \{c\}$, vP_cv' implies vP^t_cv', and (ii) for all refugees $i \in I$ and all distinct agents $v, v' \in C \cup \{i\}$, vP_iv' implies vP^t_iv'. A profile R^t breaks ties in R in favor of μ if and only if (a) R^t breaks ties in R, (b) for any landlord $c \in C$ and agent $v \in I \cup \{c\}$ with $v \neq \mu(c)$, $\mu(c)R_cv$ implies $\mu(c)P^t_cv$, and (c) for any landlord $i \in C$ and agent $v \in C \cup \{i\}$ with $v \neq \mu(i)$, $\mu(i)R_iv$ implies $\mu(i)P^t_iv$.

Lemma 1. Let μ be a matching and $R \in \mathcal{R}^V$. Then matching μ is stable at profile R if and only if there exists a strict profile $R^t \in \mathcal{R}^V$ that breaks ties in R such that matching μ is stable under R^t.

Proof. Suppose that the matching μ is stable at profile $R \in \mathcal{R}^V$ and let R^t be a strict profile that breaks ties in R in favor of μ. Then, matching μ is stable at profile R^t by construction.

In showing the converse, let R^t be a strict profile that breaks ties in R. Suppose that matching μ is stable at R^t. Then matching μ is feasible under R. Furthermore, if for some landlord-refugee pair (c, i), it holds that $iP_c\mu(c)$ and $cP_i\mu(i)$, then $iP^t_c\mu(c)$ and $cP^t_i\mu(i)$ since R^t breaks ties in R. This contradicts the assumption that matching μ is stable at profile R^t. □

Proof of Theorem 1. It needs to be demonstrated that there exists a stable maximum matching for each profile $R \in \mathcal{P}^V$ since this implies the existence of an efficient stable maximum matching for each profile $R \in \mathcal{P}^V$: because the number of stable maximum matchings is finite for each $R \in \mathcal{P}^V$, there exists a stable maximum matching which is not Pareto dominated by any other stable maximum matching, say μ is such a matching; if μ is not efficient, then there exists μ' which Pareto dominates μ, i.e. $\mu'(v)R_v\mu(v)$ for all $v \in V$. But then $|\mu'| \geq |\mu|$ and μ' must be maximum. Because μ is not Pareto dominated by any stable maximum matching, μ' must contain a blocking pair (c, i). But then (c, i) also blocks μ, a contradiction to stability of μ.

To show the existence of a stable maximum matching, let μ be a maximum matching and let R^t be a strict profile breaking ties in R in favor of μ. If matching μ is stable under R^t, then by Lemma 1, matching μ is a stable maximum matching and the proof is completed. Otherwise, there exists a blocking pair (c, i) for matching μ and a profile R^t. Note now that it cannot be the case that $\mu(i) = i$ and $\mu(c) = c$, because then μ cannot be a maximum matching. Moreover, it cannot be the case that $\mu(i) \neq i$ and $\mu(c) = c$ since refugee i then cannot strictly gain from blocking by construction of preferences R_i. Hence, it must be the case that $\mu(c) \neq c$. Since (c, i) is a blocking pair, it now follows that either:

- $q_{\mu(c)} < q_i \leq q_c$ and $l_c(i) \in A(\succeq_c)$, or;
- $q_{\mu(c)} = q_i \leq q_c$ and $l_c(i) \succ_c l_c(\mu(c))$.

Consider next matching μ' where $\mu'(c) = i$, $\mu(i) = c$, $\mu'(\mu(c)) = \mu(c)$, and $\mu'(v) = \mu(v)$ for all $v \in V \setminus \{c, i, \mu(c)\}$. Then $|\mu'| \geq |\mu|$ and μ' is a maximum matching. Let $R^{t\prime}$ be a strict profile breaking ties in R in favor of μ'. If μ' is stable under $R^{t\prime}$, then the proof is again completed by Lemma 1. Otherwise one can continue as in the above and improve the landlords' rankings and, at some point, a stable maximum matching must be found. $\square$

Proof of Theorem 2. It needs to be established that any matching $\mu \in \mathcal{V}(g,w)$ is an efficient stable maximum matching for the weighted graph $(g,w) = (V,E,R,w)$ whenever $R \in \mathcal{P}^V$ and the vector of edge weights w is defined by equation (1).

To obtain a contradiction, suppose first $\mu \in \mathcal{V}(g,w)$ but μ not is maximum. This means that there exists some other matching $\mu' \in \mathcal{A}(R)$ with $|\mu'| > |\mu|$ or, equivalently, that:

$$\textstyle\sum_{c\in\mu'(C)} q_{\mu'(c)} > \sum_{c\in\mu(C)} q_{\mu(c)}. \tag{2}$$

Because $\mu \in \mathcal{V}(g,w)$ it follows that $S(g,w,\mu) \geq S(g,w,\mu')$. By definition of the edge weights in equation (1), the latter inequality can be rewritten as:

$$\textstyle\sum_{c\in\mu(C)}(|C|oq_{\mu(c)}+(1+|L_c|-r_c(\succeq_c,l_c(\mu(c))))) \geq \sum_{c\in\mu'(C)}(|C|oq_{\mu'(c)}+(1+|L_c|-r_c(\succeq_c,l_c(\mu'(c))))),$$

or, equivalently, as:

$$\begin{aligned} |C|o\left(\textstyle\sum_{c\in\mu(C)} q_{\mu(c)} - \sum_{c\in\mu'(C)} q_{\mu'(c)}\right) &+ \textstyle\sum_{c\in\mu(C)}(1+|L_c|-r_c(\succeq_c,l_c(\mu(c)))) \\ &- \textstyle\sum_{c\in\mu'(C)}(1+|L_c|-r_c(\succeq_c,l_c(\mu'(c)))) \geq 0. \end{aligned} \tag{3}$$

The expression to the far left in condition (3) must be smaller than or equal to $-|C|o$ by condition (2). Hence, condition (3) can only hold if:

$$\textstyle\sum_{c\in\mu(C)}(1+|L_c|-r_c(\succeq_c,l_c(\mu(c)))) - \sum_{c\in\mu'(C)}(1+|L_c|-r_c(\succeq_c,l_c(\mu'(c)))) \geq |C|o. \tag{4}$$

To obtain the desired contradiction, it will be demonstrated that inequality (4) cannot hold. To see this, note first that the largest value that $(1+|L_c|-r_c(\succeq_c,l_c(\mu(c))))$ can take is $(1+o-1) = o$. Hence, even if all landlords in C are matched at μ', the largest value that the left hand side of inequality (4) can take is $|C|o$ which can only be obtained when $\mu(C) = \emptyset$. If $\mu(C) = \emptyset$, then $S(g,w,\mu) = 0$ and μ cannot be a maximum wight matching as $\mu' \in \mathcal{A}(R)$ and $S(g,w,\mu') > 0$. Thus, the value of the left hand side of (4) is smaller than $|C|o$, which is a contradiction. Hence, μ must be a maximum matching.

To demonstrate that μ is a stable matching, suppose that it is not. This means that there exists a landlord-refugee pair (c,i) at matching μ such that $iP_c\mu(c)$ and $cP_i\mu(i)$. By construction of preferences R_i, these conditions can only hold if $q_i \geq q_{\mu(c)}$ and $\mu(i) = i$. But then $iP_c\mu(c)$ contradicts that $\mu \in \mathcal{V}(g,w)$. To see this, consider the matching μ'' where $\mu''(c) = i$, $\mu''(i) = c$, $\mu''(\mu(c)) = \mu(c)$, and $\mu''(v) = \mu(v)$ for all $v \in V \setminus \{c,i,\mu(c)\}$. From the construction of the edge weights in equation (1), it now follows that $S(g,w,\mu'') > S(g,w,\mu)$, i.e., a contradiction to $\mu \in \mathcal{V}(g,w)$. Hence, matching μ must be stable.

Finally, to demonstrate that matching μ is efficient, suppose that μ is Pareto dominated by some other matching $\mu' \in \mathcal{A}(R)$. By construction of preferences R_c and by definition of efficiency, it follows that if $c \in \mu(C)$ then $c \in \mu'(C)$, i.e., if landlord c is matched at μ but unmatched at μ', then the landlord c is worse off at μ' than at μ which contradicts that μ' Pareto dominates μ. Using identical arguments, it follows that if $i \in \mu(I)$ then $i \in \mu'(I)$. Because μ is a maximum matching, by the above conclusion, it then follows that $\mu(I) = \mu'(I)$ and, consequently, that $\mu(C) = \mu'(C)$. Note next that each landlord in $\mu(C)$ is matched to a refugee with the same capacity at both μ and μ'. This follows from Assumption 3 and the conclusion that $\mu(C) = \mu'(C)$ and $\mu(I) = \mu'(I)$ together with the assumption that μ is Pareto dominated by μ'. But this conclusion and the assumption that μ is Pareto dominated by μ' imply that each landlord $c \in \mu(C)$ weakly prefers the language spoken by refugee $\mu'(c)$ to the language spoken by refugee $\mu(c)$. That is:

$$|L_c| - r_c(\succeq_c, l_c(\mu'(c))) \geq |L_c| - r_c(\succeq_c, l_c(\mu(c))) \text{ for each } c \in \mu(C). \tag{5}$$

Note also that because $\mu(I) = \mu'(I)$ and by construction of preferences R_c, it must be the case that at least one inequality in condition (5) is strict since μ is Pareto dominated by μ' by assumption. Since $\mu \in \mathcal{V}(g, w)$, it follows that:

$$\textstyle\sum_{c\in\mu(C)}(|C|oq_{\mu(c)}+(1+|L_c|-r_c(\succeq_c, l_c(\mu(c)))) \geq \sum_{c\in\mu'(C)}(|C|oq_{\mu'(c)}+(1+|L_c|-r_c(\succeq_c, l_c(\mu'(c)))).$$

Because $\mu(C) = \mu'(C)$ and $\mu(I) = \mu'(I)$, the above inequality can be simplified to:

$$\textstyle\sum_{c\in\mu(C)}(|L_c| - r_c(\succeq_c, l_c(\mu(c)))) \geq \sum_{c\in\mu(C)}(|L_c| - r_c(\succeq_c, l_c(\mu'(c)))). \tag{6}$$

But this inequality cannot hold since condition (5) holds for each $c \in \mu(C)$ with at least one strict inequality. Hence, μ must be efficient. $\square$

Proof of Proposition 2. Throughout the proof, it is assumed that $R_j = R'_j$ for all $j \in V \setminus \{i\}$ and that q_i is identical at profiles R and R'. Let now $f_i(R)$ be the match of refugee family i at matching $f(R)$ and suppose, to obtain a contradiction, that refugee family i can manipulate f by reporting $L'(i) \neq L(i)$. This misrepresentation generates a profile $R' = (R'_i, R_{-i}) \in \mathcal{P}^V$. But then $f_i(R')P_i f_i(R)$ since refugee family i can manipulate the mechanism by assumption. Since f breaks ties consistently, we may assume $L'(i) = \{l_{f_i(R')}(i)\}$.[24]

Note next that $f_i(R) = i$ and both $f_i(R') \neq i$. This follows by construction of preferences R_i and the assumption $f_i(R')P_i f_i(R)$. From the fact that $f(R) \in \mathcal{A}(R)$ and $R_j = R'_j$ for all $j \in V \setminus \{i\}$, it then follows that $f(R) \in \mathcal{A}(R')$. Identical arguments together with the fact $f(R') \in \mathcal{A}(R')$ imply $f(R') \in \mathcal{A}(R)$. Because f is a maximum matching mechanism, for R it follows $|f(R)| \geq |f(R')|$ and for R' it follows $|f(R')| \geq |f(R)|$. Hence, $|f(R')| = |f(R)|$.

[24] If not let $L''(i) = \{l_{f_i(R')}(i)\}$ and $R'' = (R''_i, R_{-i})$ be the resulting profile. Then it is easy to see $f(R') \in \mathcal{X}(R'') \subseteq \mathcal{X}(R')$. Since f breaks ties consistently, we have $f(R'') = f(R')$.

But now it follows that $f(R') \in \mathcal{X}(R)$: if $f(R')$ is not stable under R, then for any blocking pair (c, j) it must hold $j \neq i$ and (c, j) would also block $f(R')$ under R', a contradiction; and if $f(R')$ is not efficient under R, then by $L'(i) = \{l_{f_i(R')}(i)\}$, $f(R')$ is not efficient under R', a contradiction. Thus, $f(R') \in \mathcal{X}(R)$. Similar arguments (and using $L'(i) = \{l_{f_i(R')}(i)\}$) establish that $f(R) \in \mathcal{X}(R')$. Now $f(R), f(R') \in \mathcal{X}(R) \cap \mathcal{X}(R')$, which is a contradiction to the fact that f breaks ties consistently. $\square$

Proof of Proposition 3(i). This part is proved using a simple example. Let $C = \{c_1, c_2, c_3\}$ and $I = \{1, 2, 3\}$, and suppose that $q_v = 1$ for all $v \in V$. The induced preference profile R is given by the below table (it is easy to verify that there exists lists $(\succeq_{c_1}, \succeq_{c_2}, \succeq_{c_3})$ and $(L(1), L(2), L(3))$ that are consistent with profile R).

R_{c_1}	R_{c_2}	R_{c_3}	R_1	R_2	R_3
1	2	2	c_1, c_3	c_2, c_3	c_1, c_2
3	3	1			

The set $\mathcal{V}(g, w)$ contains only the following two matchings:

$$\mu = \begin{pmatrix} c_1 & c_2 & c_3 \\ 1 & 3 & 2 \end{pmatrix} \text{ and } \mu' = \begin{pmatrix} c_1 & c_2 & c_3 \\ 3 & 2 & 1 \end{pmatrix}.$$

The interpretation of matching μ is that landlord c_1 is matched to refugee 1, landlord c_2 is matched to refugee 3, and landlord c_3 is matched to refugee 2. From matching μ and μ', it follows that $f_{c_1}(R) = 3$ or $f_{c_2}(R) = 3$. The choice depends on how f breaks ties consistently.

Suppose first that $f_{c_1}(R) = 3$ and that landlord c_1 misrepresents preferences over acceptable languages by reporting $\succeq'_{c_1}$ such that $s_1 \succ'_{c_1} c_1$. Denote the new preference profile by (R'_{c_1}, R_{-c_1}). In this case, μ is the unique maximum matching for profile (R'_{c_1}, R_{-c_1}) and, consequently, $f(R'_{c_1}, R_{-c_1}) = \mu$. But then $f_{c_1}(R'_{c_1}, R_{-c_1}) P_{c_1} f_{c_1}(R)$, which means that landlord c_1 has a profitable deviation from R.

Suppose next that $f_{c_2}(R) = 3$ and that landlord c_2 misrepresents preferences over acceptable languages by reporting $\succeq'_{c_2}$ such that $s_2 \succeq'_{c_2} c_2$. Denote the new preference profile by (R'_{c_1}, R_{-c_1}). In this case, μ' is the unique maximum matching for profile (R'_{c_2}, R_{-c_2}) and, consequently, $f(R'_{c_2}, R_{-c_2}) = \mu'$. But then $f_{c_2}(R'_{c_2}, R_{-c_2}) P_{c_2} f_{c_2}(R)$, which means that landlord c_2 has a profitable deviation from R. $\square$

Proof of Proposition 3(ii). Throughout, let $R_j = R'_j$ for all $j \in V \setminus \{c\}$ and that $\succeq_c$ is identical at profiles R and R'. Let now $f_c(R)$ be the match of landlord c at matching $f(R)$ and suppose, to obtain a contradiction, that landlord c can manipulate f by reporting $q'_c \neq q_c$. This misrepresentation generates a profile $R' = (R'_c, R_{-c}) \in \mathcal{P}^V$. Hence, $f_c(R') P_c f_c(R)$ since landlord c can manipulate f by assumption.

Note that landlord c cannot be matched to a refugee family with strictly more family members than q_c at matching $f(R')$, i.e., an unacceptable refugee family, and this contradicts that $f_c(R') P_c f_c(R)$ by construction of preferences R_c. Thus, $q_{f_c(R')} \leq q_c$.

Note next that landlord c cannot be matched to a refugee family with strictly fewer family members at matching $f(R')$ than at matching $f(R)$ since this contradicts that $f_c(R')P_c f_c(R)$ by construction of preferences R_c. Thus, $q_{f_c(R)} \leq q_{f_c(R')} \leq \min\{q'_c, q_c\}$. But now we have both $f(R), f(R') \in \mathcal{A}(R)$ and $f(R), f(R') \in \mathcal{A}(R')$. Because only the capacity q_c changes to q'_c from R to R', now it is straightforward that $f(R), f(R') \in \mathcal{X}(R) \cap \mathcal{X}(R')$ contradicts the fact that f breaks ties consistently. $\square$

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